HISTOGRAMS ON LINGUISTIC COLOR VARIABLES BASED ON FUZZY NATURALS

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Abstract

In this paper we introduce a new fuzzy histogram approach based on fuzzy naturals. In our proposal, histograms are functions on a fuzzy color space, where the occurrences of a given fuzzy color are counted by means of a new definition of fuzzy natural number (represented by means of a probability distribution on $\mathbb{N}$). This approach to histograms avoids the well-known disadvantages of the ordinary sigma-count. We illustrate the potential application of the proposal by applying it to the problem of dominant color selection.

Keywords: Fuzzy colors, Fuzzy histograms.

1 INTRODUCTION

In some image processing applications it is important to take into account the particular perception of colors of a certain user. However, there is a semantic gap between the human perception of colors and the data stored in a pixel, i.e., the three (or more) components corresponding to the primary colors in a given color space (e.g., red green and blue for RGB). To face this problem, several authors have proposed to employ fuzzy sets [10, 11]. In these approaches, colors in an image are linguistic labels represented as fuzzy subsets of triplets of a certain color space [19, 21]. This is important in order to determine the semantic interpretation of colors for human-machine interaction, or to take into account the similarity between colors in a color space as perceived by the user [18].

In the scope of image analysis and processing, the histogram is the basis for numerous techniques for image restoration, enhancement, segmentation, retrieval, etc [16].

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The histogram of an image is a function $h(\mathbf{c_k}) = n_k$ where $\mathbf{c_k} = [x, y, z]$ is a color and n_k is the number of pixels in the image having the color $\mathbf{c_k}$. It is common to normalize a histogram by dividing each of its values by the total number of pixels, obtaining an estimate of the probability of occurrence of a color $\mathbf{c_k}$.

Working with fuzzy colors suggests to extend the notion of histogram to ”fuzzy histogram” in order to manage the imprecision in (i) color description by means of fuzzy colors and, consequently, in (ii) fuzzy color counting. In this sense, a fuzzy histogram will give us information about the frequency of each fuzzy color.

In the literature there are several proposals which define histograms over a set of fuzzy colors [6, 9, 15]. One drawback of most of these proposals is that they work only with intensities. In addition, the counting of fuzzy colors is performed by using the sigma-count (i.e., the sum of membership degrees). However, the sigma-count is not a suitable measure of cardinality in many applications, as it has been recognized by several authors [8, 12]. Proposals based on the sigma-count summarize the counting in a single number, so they do not represent the imprecision of the count itself.

To solve the problems above, after introducing a formalization of the concept of fuzzy color space, in this paper we propose to ”count” the occurrences of a given fuzzy color by means of the new definition of fuzzy natural numbers introduced in [4]. In this definition, a natural number is represented by means of a probability distribution on $\mathbb{N}$.

The rest of the paper is organized as follows. Section 2 formalize the concepts of fuzzy color and fuzzy color space. In section 3 the fuzzy natural numbers are introduced. On the basis of the fuzzy naturals, the fuzzy histogram over fuzzy colors is defined in section 4. An application to dominant color selection is presented in section 5 to illustrate the potential applications and performance of the new definition of fuzzy histogram. Some examples are showed in section 6 and, finally, the main conclusions are summarized in section 7.

2 FUZZY MODELLING OF COLORS

In this section, the notions of fuzzy color (section 2.2) and fuzzy color space (section 2.3) are introduced as an extension of the classical concepts of color and color space (section 2.1).

2.1 COLOR AND COLOR SPACES

For representing colors, several color spaces can be used. In essence, a color space is a specification of a coordinate system and a subspace within that system where each color is represented by a single point. The most commonly used color space in practice is RGB because is the one employed in hardware devices (like monitors and digital cameras). It is based on a cartesian coordinate system, where each color consists of three components corresponding to the primary colors red, green, and blue.

Nevertheless, it is well known that RGB it is not always the most adequate space for color image analysis. Furthermore, the color components of this space do not have an intuitive interpretation according to the human perception of color. Instead, other color spaces based on human perception (HSI, HSV or HSL) seem to be a better choice [16]. In these perceptual spaces, the hue component (H) represents the color tone (for example, red or blue), saturation (S) is the amount of color (for example, vivid red or pale red) and the third component (called intensity, value or lightness) is the amount of light (it allows to distinguish between a dark color and a light color). Therefore, if linguistic labels are needed for color description, this kind of color spaces are more suitable than RGB, linear combination of RGB (like CMY, YCbCr, YUV, etc.) or perceptually uniform color spaces (like CIELa*b*, CIELuv, etc.).

2.2 FUZZY COLOR

In order to manage the imprecision in color description, we introduce the following definition of fuzzy color:

Definition 2.1 *A fuzzy color $\widetilde{C}$ is a normalized fuzzy subset of colors.*

As previously explained, colors can be represented as a triplet of real numbers corresponding to coordinates in a color space. Hence, a fuzzy color can be defined as a normalized fuzzy subset of points of a color space. From now on, we shall note XYZ a generic color space with components X, Y and Z[1], and we shall assume that a color space XYZ, with domains D_X, D_Y and D_Z of the corresponding color components is employed. This leads to the following more specific definition:

[1] Although we are assuming a three dimensional color space, the proposal can be easily extended to color spaces with more components.

Definition 2.2 *A fuzzy color $\widetilde{C}$ is a linguistic label whose semantics is represented in a color space XYZ by a normalized fuzzy subset of $D_X \times D_Y \times D_Z$.*

Notice that it is possible to represent the same fuzzy color by means of different fuzzy subsets on different color spaces, provided they have the same expressive power. For example, a fuzzy color *red* could be represented as a fuzzy subset with semantics “approximately [255,0,0]” if we employ the RGB color space and also as a fuzzy subset with semantics “approximately [0,1,128]” if we use the HSV space.

As in the case of colors, that can be described by means of three precise component values, it is possible (though not obligatory) to define a fuzzy color by means of fuzzy subsets on the domain of each component. We introduce the following definition:

Definition 2.3 *A fuzzy color component $\widetilde{C}_X$ (resp. $\widetilde{C}_Y$, $\widetilde{C}_Z$) is a linguistic label whose semantics is represented by a normalized fuzzy subset of D_X (resp. D_Y, D_Z).*

By using these concepts, a fuzzy color $\widetilde{C}$ can be defined and represented in practice by a triple $[\widetilde{C}_X, \widetilde{C}_Y, \widetilde{C}_Z]$, where $\widetilde{C}_X$, $\widetilde{C}_Y$, and $\widetilde{C}_Z$ are fuzzy color components of $\widetilde{C}$. This way, the fuzzy subset representing a fuzzy color can be obtained by combining the corresponding fuzzy color components in a suitable way. Thus, for every crisp color $[x, y, z]$, its membership degree to $\widetilde{C}$ is defined as

$$\widetilde{C}(x, y, z) = \bigwedge\{\widetilde{C}_X(x), \widetilde{C}_Y(y), \widetilde{C}_Z(z)\} \qquad (1)$$

with $\bigwedge$ being a t-norm, usually the minimum. From now on, we shall note $I_{\widetilde{C}}$ the fuzzy subset of pixels with color $\widetilde{C}$

2.3 FUZZY COLOR SPACE

For extending the concept of color space to the case of fuzzy colors, and assuming a fixed color space XYZ, with D_X, D_Y and D_Z being the domains of the corresponding color components, the following definition is introduced:

Definition 2.4 *A fuzzy color space $\widetilde{XYZ}$ is a set of fuzzy colors that define a partition of $D_X \times D_Y \times D_Z$.*

As in the case of single colors, one very convenient way of defining and representing a fuzzy color space is to employ a fuzzy space for each component. We introduce this idea in the following definition:

Definition 2.5 *A fuzzy space over the component X (resp. Y, Z) is a set of fuzzy color components that define a partition of D_X (resp. D_Y, D_Z).*

This procedure has several advantages. First, less linguistic labels have to be defined. Second, we can represent and work with every component of the fuzzy color individually (this is specially interesting in perceptual color spaces like HSI where, for example, we could query for colors with red hue). Finally, the linguistic labels associated to fuzzy colors can be obtained by combining the corresponding linguistic labels of each component (for example, in the case of HSI color space, the color label "red-saturated-bright" is obtained by combining hue, saturation and intensity labels).

3 FUZZY NATURALS

In order to obtain an histogram, which give us information about the frequency of each color in the image, we need to count the number of pixels having a particular color. In our case, we must count pixels having fuzzy colors. For this purpose we employ the concepts of fuzzy natural.

In this section we explain the definition of fuzzy naturals introduced in [4] (section 3.2). For defining fuzzy naturals the definition of cardinality of a fuzzy set will be used (section 3.1)

3.1 FUZZY CARDINALITY

The definition of fuzzy natural can be obtained directly from the definition of cardinality of a fuzzy set. In the crisp case, as it is well known, naturals are the possible cardinalities of crisp sets. The usual (in the sense of most employed) way to extend cardinality to the fuzzy case is the scalar sigma-count, that can defined for any fuzzy set F with membership function $F : X \to [0, 1]$ as

$$sc(F) = \sum_{x \in X} F(x) \qquad (2)$$

However, the sigma-count is not a suitable measure of cardinality in many applications, as it has been recognized by several authors [8, 12]. In the particular problem of defining fuzzy naturals, sigma-count is counterintuitive, since sc is not a natural number in general. Even if the result is a natural number, we can obtain the same result in very different situations, thus loosing information. This is the case if we consider two situations like having 100 pixels compatible with a color $\widetilde{C}$ to a degree 0.1, and having 10 pixels compatible with $\widetilde{C}$ to a degree 1; in both cases, the result of the sigma count applied to the fuzzy set of pixels compatible with $\widetilde{C}$ is 10. On the basis of these problems, it is widely accepted that the best way to represent the cardinality of a fuzzy set is by means of a fuzzy subset of the naturals [3, 7].

Most of the approaches consider that such fuzzy subsets of the naturals must be convex. However, in [3] we showed that in some cases this is counterintuitive. Consider for example the fuzzy set given by $A = 1/x_1 + 0.5/x_2 + 0.5/x_3$. The cardinality of A could be one (because x_1 belongs to A for sure) or, if we relax our criterion to accept elements in A, the cardinality could be three (accepting x_2 and x_3 belong to A as well). However, the cardinality cannot be two, since if $x_2 \in A$ then $x_3 \in A$ and vice versa. This way, the cardinality is not convex. In addition, this example illustrates that the sigma-count is not always a good measure since $sc(A) = 2$. In summary, sc is not a good cardinality approach for the definition of fuzzy natural numbers since it is a real number and can give counterintuitive results.

Several authors [3, 13, 14] have pointed out that the possible cardinalities of a fuzzy set are the cardinalities of its α-cuts, since these are the possible crisp representatives of the fuzzy set. In our previous example the possible cardinalities of A are 1 or 3 since its possible α-cuts are $\{x_1\}$ and $\{x_1, x_2, x_3\}$. In general, this is in accordance with the principle that there is a single membership scale for fuzzy sets, i.e., $F(x) = F(x')$ means that x is F just like x', so we cannot consider $x \in F$ and $x' \notin F$ at the same time, and either both or none of them contribute to the cardinality of F.

As we have just seen, the sigma-count does not necessarily correspond to the cardinality of an α-cut ($sc(A) = 2$). On the contrary, there are two fuzzy cardinalities that comply with this idea, calculating the possible cardinalities of a fuzzy set as the cardinalities of its α-cuts, and assigning to each cardinality a degree of representativity, as we detail in the following subsections:

3.1.1 Zadeh's first fuzzy cardinality

Zadeh introduced in 1979 a non-convex fuzzy measure for the cardinality of a fuzzy set F to be [20]

$$\mu_{CARD(F)}(k) = sup\{\alpha \mid |F_\alpha| = k\} \qquad (3)$$

For example, for $A = 1/x_1 + 0.5/x_2 + 0.5/x_3$ we have $\mu_{CARD(A)} = 1/1 + 0.5/3$. In general, $\mu_{CARD(\cdot)}$ is a fuzzy subset of $\mathbb{N}$ verifying it is normal, its core consists of one value only, and it is strictly decreasing in its support.

3.1.2 The Fuzzy Cardinality ED

In [3] the fuzzy cardinality of a fuzzy set F is defined as

$$ED(F) = \sum_{\alpha_i \in \Lambda(F)} (\alpha_i - \alpha_{i+1}) / |F_{\alpha_i}| \tag{4}$$

where $\Lambda(F) = \{\alpha_1, \ldots, \alpha_q\} = \{F(x_i) \mid x_i \in supp(F)\} \cup \{1\}$ is the set of representative α-cuts of F, with $1 = \alpha_1 > \alpha_2 > \cdots > \alpha_q > \alpha_{q+1} = 0$. For example, $\Lambda(A) = \{1, 0.5\}$ and $ED(A) = 0.5/1 + 0.5/3$.

The measure ED is a probability distribution on $\mathbb{N}$. It is possible to show that the converse is also true, i.e., any probability distribution on $\mathbb{N}$ is the cardinality of some fuzzy set as given by ED. The proposition and the corresponding proof can be found in [4].

In [3] it is shown that ED is the basic probability assignment of the possibility distribution $\mu_{CARD(\cdot)}$, and hence both cardinalities are isomorphic. However, ED has been shown to be more appropriate than $\mu_{CARD(\cdot)}$ when using the fuzzy cardinality in practice, for example in the evaluation of quantified sentences [5].

3.2 FUZZY NATURAL DEFINITION

The idea that cardinality and natural numbers are basically identical was employed in [4] to define fuzzy natural numbers as follows:

Definition 3.1 *A fuzzy natural N is any probability distribution on $\mathbb{N}$ of the form*[2]

$$N = \sum_{n_i \in \mathbb{N}} p_i / n_i \tag{5}$$

with $\sum_{n_i \in N} p_i = 1$.

As an example, let us consider the fuzzy natural

$$N = 0.2/0 + 0.6/2 + 0.2/4 \tag{6}$$

Having N objects means that the number of objects is 0 with probability 0.2, 2 with probability 0.6, and 4 with probability 0.2.

In the context of histograms, this definition allows us to count the number of pixels that comply with a certain fuzzy color $\widetilde{C}$ as a fuzzy natural, corresponding to the cardinality of the fuzzy subset of pixels whose color is $\widetilde{C}$. This fuzzy natural can be understood as a (fuzzy) estimation of the counting of $\widetilde{C}$ in the image.

[2]Notice that we are using in the definition of N the usual notation $\sum$ for fuzzy sets on a discrete referential, while in $\sum_{n_i \in N} p_i = 1$, $\sum$ means addition.

4 FUZZY HISTOGRAMS

We define a fuzzy histogram as follows:

Definition 4.1 *Let $\widetilde{XYZ}$ be a fuzzy color space. A fuzzy histogram is a function h that assigns a fuzzy natural to every fuzzy color in $\widetilde{XYZ}$, i.e.,*

$$h(\widetilde{C}) = N_{\widetilde{C}} \tag{7}$$

with $N_{\widetilde{C}}$ a fuzzy natural.

Notice that this definition extends to the fuzzy case the idea of histogram as a function that assign a count, given by a natural number, to every color. In the fuzzy context, we have fuzzy colors (instead crisp ones) and fuzzy naturals for imprecise counting.

Fuzzy histograms are descriptions of the fuzzy counting for a pixel having a color $\widetilde{C}$, for every color in a fuzzy color space. However, if this is the final information to be provided to a user, it is convenient to summarize it so that it is easier to understand. Of course, any possible summary causes a loss of information. We can consider several options as a summary of the fuzzy frequency of a fuzzy color:

- Linguistic summary of the fuzzy naturals. In this option, a triangular fuzzy number (in the classical sense of "possibility distribution on the set or real numbers") $\mathcal{R}_{\widetilde{C}}$ is calculated as an approximation of the fuzzy natural $N_{\widetilde{C}}$. The number is obtained by approximation techniques as the number that yields a maximum accomplishment degree with $N_{\widetilde{C}}$ with minimum entropy. This technique is described in [1].

- The interval of natural numbers $[a, b]$ with $a = \min\{i \in support(N_{\widetilde{C}})\}$ and $b = \max\{i \in support(N_{\widetilde{C}})\}$

- The natural number with higher probability in the probability distribution $N_{\widetilde{C}}$. Notice that this number, being an scalar, is obtained from the cardinality of an α-cut of $I_{\widetilde{C}}$, and in this sense is coherent with the ideas explained at the beginning of section 3.

- The natural number l obtained as the cardinality of the 0.5-cut of the fuzzy subset of pixels $I_{\widetilde{C}}$. This scalar value is the definition of approximation of the cardinality of a fuzzy set proposed by Ralescu [13]. In addition, if the fuzzy color space is a partition in the sense of Ruspini, the 0.5-cut corresponds to a crisp partition of the color space, and the values obtained for all the fuzzy colors correspond to a crisp histogram based on this crisp partition.

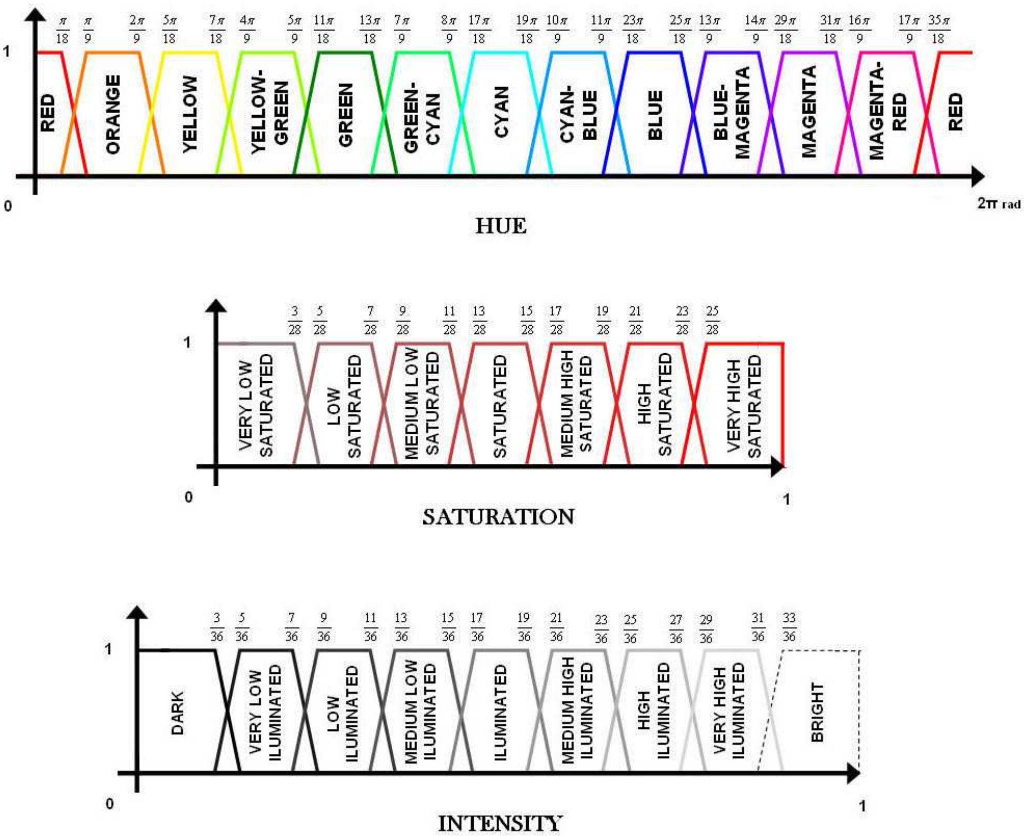

Figure 1: The fuzzy HSI color space used in the experiments.

Let us remark once more that the latter are different ways to obtain a summary that can be useful for a better understanding of the fuzzy histogram by an user, at the cost of loosing information. However, if the histogram is just an intermediate step in a calculation, we can extend the operations to the fuzzy case by operating on α-cuts and then summarizing the final result [4]. This option preserves all the information until the final step. The option of extending the operations to the fuzzy case by operating on one of the previously suggested summaries discards many information, and it is reasonable in our opinion only when time restrictions force us to obtain a less accurate, but faster, calculation.

Notice that we are not suggesting that the calculations performed by α-cuts are necessarily slow; the idea is that performing a certain operation in every α-cut takes more time than performing the same operation on a single scalar approximation. Depending on the time expended by each operation and the number of α-cuts employed (the latter can be fixed in order to obtain an approximate, but more accurate, result; a typical value is 20), we may choose one or another alternative.

5 APPLICATION TO DOMINANT COLOR SELECTION

In this section we illustrate the potential applications and performance of the new definition of fuzzy histogram by applying it to select a set of dominant fuzzy colors. Dominant colors arise as a powerful tool for describing the representative colors in an image. In fact, they are a very efficient tool for retrieving images in large databases.

Intuitively, a color is dominant to the extent it appears frequently in a given image. Therefore, the use of fuzzy histograms comes up as a suitable tool for finding the dominant fuzzy colors.

Let h be a fuzzy histogram for a color image on a fuzzy color space $\widetilde{XYZ}$. Given a color $\widetilde{C}$, its fuzzy frequency is given by $h(\widetilde{C}) = N_{\widetilde{C}}$ with $N_{\widetilde{C}} = \sum_{n_i \in \mathbb{N}} p_i/q_i$ being a fuzzy natural number in the sense of definition 3.1. We define the degree of dominance of the fuzzy color $\widetilde{C}$ as

$$Dom(\widetilde{C}) = \sum_{q_i > T} p_i \tag{8}$$

with $T \in (0, 1]$ being a threshold for selecting significant frequencies.

As set of dominant fuzzy colors we will select those ones with higher dominant degree (we can fix a number of dominant colors or select those ones which dominant degrees are greater than a threshold)

6 RESULTS

In this section we show the results obtained by applying the proposed technique to several examples. Firstly, the fuzzy histogram will be calculated over several images. Secondly, these histograms will be used to select dominant fuzzy colors.

In our experiments, the HSI color space have been used. To define the fuzzy color space $\widetilde{HSI}$ we have employed the fuzzy spaces for hue, saturation, and intensity proposed in [2] that are shown in figure 1. This proposal used as reference the Munsell color space [17] which divided in 12, 7 and 9 intervals the hue, saturation, and intensity respectively. Each one of these intervals are fuzzified using a trapezoidal function to define the memberships functions, obtaining a fuzzy partition in the sense of Ruspini.[3]

Figure 2 shows two images, the first one containing eight colors, each one compatible with degree 1 with a fuzzy color obtained by combining the fuzzy saturation label "High saturated" and the fuzzy intensity label "Illuminated", with eight different fuzzy hue labels, specifically all the hue labels except "Yellow-Green", "Cyan", "Blue-Magenta", and "Red" in figure 1. In the second one we have four colors compatible with the same saturation and intensity labels to degree 1. Each one of these four colors is compatible to a degree 0.5 with two of the eight fuzzy hue labels mentioned before. In both images, pixels are equidistributed among colors.

In both images, the set of fuzzy colors with probability greater than 0 is the set of eight fuzzy colors mentioned above. It is easy to see that the sigma-count histogram is the same for both images, specifically the probability is 1/8 for each fuzzy color. However, the perception of the colors that appear in the image and the corresponding frequencies is very different in both images.

On the contrary, the histogram based on our definition of fuzzy frequencies yield a different result in each case. In the first one, since we are in a crisp case, the result is the same provided by the sigma-count histogram, i.e., probability 1/8 (given as a probability distribution 1/(1/8)) for each fuzzy color. On the contrary, our approach yields for the second image a fuzzy histogram where, for each one of the eight color, the fuzzy probability is represented by the probability distribution $0.5/0 + 0.5/(1/4)$. This is in accordance with our intuition since each fuzzy color is compatible to a degree 0.5 with one of the colors that appear in the image and, as a consequence, we have a probability 0.5 that the fuzzy label appears in the image, and the same probability that it does not appear. Hence, the same probabilities are assigned to the value 1/4 in the first case, and 0 in the second case.

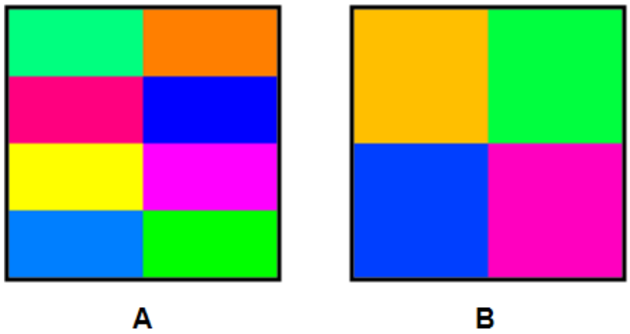

Figure 2: Multicolored images. (A) Image containing eight colors, each one compatible with degree 1 with a fuzzy color (B) Image containing four colors, each one compatible with degree 0.5 with a fuzzy color

Figures 3(B) to 3(I) show the fuzzy histogram for image 3(A). For plotting the histogram, a graph has been drawn for each intensity label. The X axis of these graphs shows the color labels ordered by hue and, within a given hue, by saturation. The Y axis shows the support of $h(\widetilde{C})$ normalized by the number of pixels n. More specifically, the graph plots pairs $(\widetilde{C}, n_i/n)$ where n_i are the crisp naturals that appear in the probability distribution $N_{\widetilde{C}}$. In addition, a dotted line is drawn between the maximum and minimum values a and b of the support, $a = \min\{i \in support(N_{\widetilde{C}})\}$ and $b = \max\{i \in support(N_{\widetilde{C}})\}$. This dotted line shows the interval $[a, b]$ that corresponds to one of the ways of summarizing $h(\widetilde{C})$, as explained in section 4. The values $N_{\widetilde{C}}(n_i)$ are not displayed.

We have compared the results obtained in this and other images with the crisp histogram for a crisp partition obtained as the set of 0.5-cuts of the fuzzy colors. Though the latter is a summary and many information is lost, we have found that higher (resp. lower) crisp frequencies in the crisp histogram correspond to higher (resp. lower) values in the support of fuzzy frequencies and, in this sense, the results are coherent with what is expected. However, the fuzzy histogram contains much more information since, contrary to the crisp one, it reflects the counting imprecision.

The fuzzy histogram shows greater frequency values in the support of the most frequent fuzzy colors in the image. For example, the highest values correspond to the fuzzy colors Black (Dark intensity) and the brown-like colors [Orange,Medium-Low-Saturated,Illuminated] and [Orange,Saturated,Illuminated], these three fuzzy colors corresponding to the background. The next colors in terms

[3] The problem of non-representability in semichromatic (undefined hue) and achromatic (undefined hue and saturation) is taken into account in the sense proposed in [2]

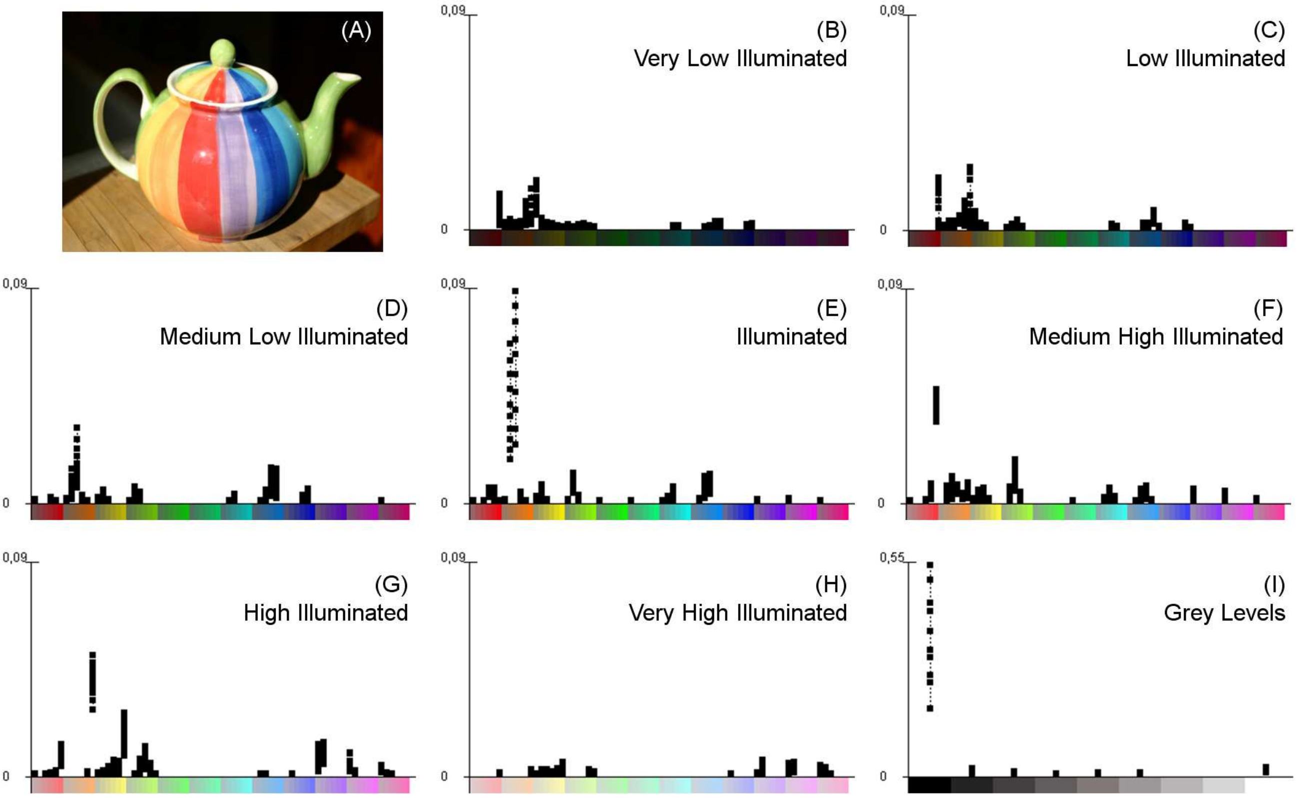

Figure 3: (A): color image. (B)-(H): fuzzy histograms for colors with different intensity labels except “Dark” and “Bright”. In each histogram, the 12x6 colors (“Very low saturated” colors, i.e., fuzzy grey levels, are not considered) are ordered first by hue and then by saturation. (I): Fuzzy colors corresponding to fuzzy grey levels.

of high frequency are those of the teapot.

Figure 4 shows the degree of dominance of those colors with degree of dominance greater than 0 for the image in Figure 3(A). The dominance has been calculated by using a threshold $T = 0.015$. The results are coherent with the information in the fuzzy histogram. With respect to our perception of dominance in the image, we must take into account that the fuzzy color space employed in the chromatic area of the HSI color space is a equidistributed fuzzy partition. However, as pointed out by several authors, we tend to employ a non-equidistributed partition in which a single fuzzy color covers approximately the support of the fuzzy colors with hue “Cyan”, “Cyan-Blue” and “Blue”. Something similar happens with the fuzzy colors with hue labels around “Green”. The consequence in this example is that the colors we see as blue are distributed between the fuzzy colors with hue “Cyan”, “Cyan-Blue” and “Blue”, so the degree of dominance of what we see as blue is maybe less than expected. On the contrary, if we consider a fuzzy color space with a label “Blue” covering the support of the three above mentioned blue-based colors, the dominance of the new label would be similar to other fuzzy colors with “Red”, “Orange” or “Yellow” fuzzy hue.

7 CONCLUSIONS

In this paper a novel definition of fuzzy histogram has been proposed. Firstly, a formalization of the concept of fuzzy color and fuzzy color space has been presented. Secondly, the occurrences of a given fuzzy color have been counted by means of a new definition of fuzzy natural number.

The proposal improves the classical sigma-count based histogram. In fact, our fuzzy histogram represents the imprecision of the count itself, contrary to sigma-count that summarizes the counting in a single number.

The performance of the proposal has been shown with several images, introducing some examples where histograms based on sigma-count are not a suitable representation of the amount of pixels painted with each color. In addition, we have illustrated the potential applications of the fuzzy histogram by applying it to the problem of dominant fuzzy colors selection.

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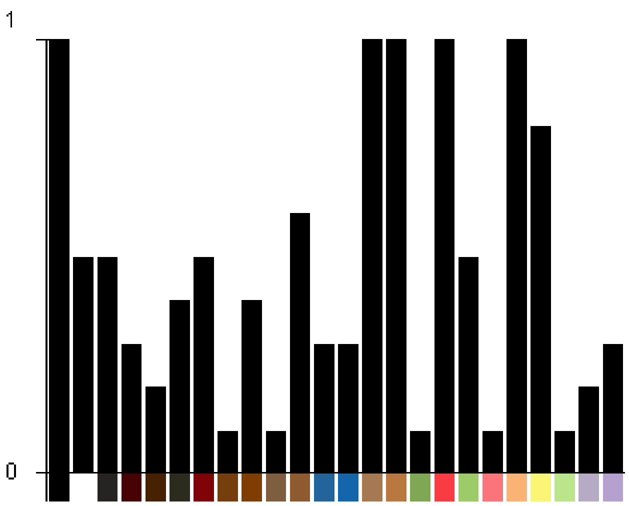

Figure 4: Degree of dominance of fuzzy colors in the image of figure 3(A). Only fuzzy colors with non-zero dominance are shown.

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