Compressive Sensing and Blind Image Deconvolution

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Abstract: In this paper, we summarize our recent results on simultaneous compressive sensing reconstruction and blind deconvolution of images, captured by a compressive imaging system introducing degradation of the captured scene by an unknown point spread function.

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1. Introduction

Compressive sensing (CS) has become a burgeoning research field due to its interesting theoretical nature and potential aid in numerous practical applications. It is based on the underlying assumption that the signal of interest is sparse in its native domain or can be sparsely approximated by a small number of nonzero coefficients in some transform domain (e.g., Fourier, Wavelet, Gradient, etc.).

Given a CS measurement matrix $\mathbf{\Phi} : M \times N$, $(M \ll N)$, whose rows correspond to the required projection vectors, and assuming that the unknown signal (e.g., image) is of size $m \times n = N$, the standard formulation of the CS acquisition process is given by,

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x} + \mathbf{n}, \tag{1}$$

where vector $\mathbf{y} : M \times 1$ corresponds to the CS observation, vector $\mathbf{x} : N \times 1$ represents the lexicographically ordered unknown signal and vector $\mathbf{n} : M \times 1$ is the measurement noise.

Most imaging devices can introduce blurring into the acquisition process, described by their point spread function (PSF). The PSF of an imaging system depends on its constituting elements and geometry (e.g., PSF of the lens, limited aperture dimensions, lack of focus, limitations of the sensor, etc.) as well as environmental parameters (e.g., atmospheric turbulence, camera motion, etc.). The introduced degradation can be represented by a blurring matrix $\mathbf{H} : N \times N$, such that vector $\mathbf{g} : N \times 1$, $\mathbf{g} = \mathbf{H}\mathbf{x}$ corresponds to the lexicographically ordered blurred version of the original signal $\mathbf{x}$. The degradation is generally nonlinear (due to saturation, quantization, etc.) and spatially varying (lens imperfections, nonuniform motion, etc.). However, most of the relevant work approximates it by a linear spatially invariant (LSI) system, where the original signal is convolved by a blurring kernel.

Even though the PSF of an imaging system can be calculated through simulations of the imaging architecture or experimental measurements, its dependence on environmental parameters implies that it cannot always be known *a priori*. Therefore, for CS imaging systems introducing degradation in the captured scene, the acquisition process is better modeled by,

$$\mathbf{y} = \mathbf{\Phi}\mathbf{g} + \mathbf{n} = \mathbf{\Phi}\mathbf{H}\mathbf{x} + \mathbf{n}. \tag{2}$$

The aforementioned model has been recently introduced in the image processing literature, mainly for the non-blind case, when $\mathbf{H}$ is known. In [1], the authors proposed this model for reducing the measurements obtained by satellites and aerospace probes of the next generation deep-space imaging systems. Extensions of [1] were presented in [2, 3]. Except for imaging applications, the model in (2) is also suitable for modeling filtered sparse processes, [4–6].

The previous analysis shows that the model introduced in (2) has numerous practical applications. However, its theoretical background is still infantile. To the best of our knowledge, for imaging applications, there is almost no related work on the CS blind deconvolution (BD) problem. We have followed two different approaches for its solution and preliminary results have been presented in [7, 8]. Extensions of these approaches are expected to appear in [9, 10].

2. Proposed Approaches

In the following, we briefly describe the main ideas behind our derived algorithms and show representative comparisons with other methods. The first approach resembles a Total Variation (TV) minimization problem, following Bayesian modeling and appropriately chosen priors for the image and blur while the second is inspired by compound ℓ_1-TV minimization schemes.

2.1. *Gradient sparsity based approach*

In our first approach, [8,9], we use an image prior based on the commonly used assumption that image derivatives are sparse while a smooth prior is used for the blur. The problem to be solved can be expressed as

$$\underset{\mathbf{x},\mathbf{h}}{\text{minimize}} \;\; \frac{\beta}{2}\|\mathbf{y}-\mathbf{\Phi}\mathbf{H}\mathbf{x}\|^2 + R_1(\mathbf{x},\mathbf{A}_k) + \gamma R_2(\mathbf{h}) \tag{3}$$

where $R_1(\mathbf{x},\mathbf{A}_k)$ represents the sparse image prior which is controlled by a set of regularization parameters a_{ki} using $k = 1\ldots L$ high-pass filters per image pixel $i = 1\ldots N$, such that $\mathbf{A}_k = diag(a_{ki})$ and $R_2(\mathbf{h})$ is a smooth blur prior. The image prior is essentially a TV prior with additional degrees of freedom.

Using Bayesian inference and the *Expectation-Maximization* procedure we devised an algorithm for simultaneous estimation of the unknown sharp image $\mathbf{x}$, the unknown blur $\mathbf{h}$ and all modeling parameters a_{ki}, β and γ.

2.2. *Transformation domain sparsity based approach*

Blurred images can be shown to be compressible, [10]. Indeed, blurring of an image usually results in higher sparsity in some transform domains (e.g., Fourier, Wavelet). Such behavior is expected since blurring constitutes a low-pass filtering operator. We denote a basis by $\mathbf{W} : N \times N$ and a sparse vector of transformed coefficients by $\mathbf{a} : N \times 1$, such that a blurred image can be represented as $\mathbf{Hx} = \mathbf{Wa}$.

We proposed a general constrained optimization problem which allows us to recover, simultaneously, the transformed coefficients $\mathbf{a}$ of the blurred image, the unknown image $\mathbf{x}$, and the unknown blur $\mathbf{h}$,

$$\begin{aligned} &\underset{\mathbf{x},\mathbf{h},\mathbf{a}}{\text{minimize}} \;\; \frac{\beta}{2}\|\mathbf{y}-\mathbf{\Phi}\mathbf{W}\mathbf{a}\|^2 + \tau\|\mathbf{a}\|_1 + \alpha R_1(\mathbf{x}) + \gamma R_2(\mathbf{h}) \\ &\text{subject to } \mathbf{Hx} = \mathbf{Wa}. \end{aligned} \tag{4}$$

where $R_1(\mathbf{x})$ and $R_2(\mathbf{h})$ are application specific regularization functionals and α, β, γ, τ are appropriately chosen modeling parameters.

The advantage of (4) compared to a sequential CS BD approach is the ability to impose an additional structural constraint on the transformed coefficients $\mathbf{a}$. We accomplish this through the relation $\mathbf{Hx} = \mathbf{Wa}$ and by exploiting the convolutional inter-dependencies between the image $\mathbf{x}$ and the blur $\mathbf{h}$ when solving for the transformed coefficients $\mathbf{a}$. The minimization problem in (4) can be solved through *Alternating Maximization* using the quadratic penalty method [7, 10] or through the augmented Lagrangian method of multipliers which we are currently exploring.

3. Experimental Results

We have performed a large set of simulations with various images under different blur degradations, noise realizations and CS ratios. Based on our experimental results both approaches outperform current non-blind deconvolution algorithms and their blind counterparts achieve reasonable performance. The first approach appears to be more suitable for images with lower sparsity exhibiting multiple edges while the second usually favors solutions with higher sparsity or piecewise-smooth signals.

Figure 1 presents example blind restoration results using the first approach for four different images degraded with a Gaussian PSF of variance 5 and signal to noise ratio (SNR) 40 dB. Figure 2 presents an indicative average performance comparison of our second approach with a set of non-blind deconvolution algorithms, whose references are not included due to space limitations, but their codes are available online. The employed comparison metric is the peak signal to noise ratio, defined as $PSNR = 10\log_{10}(NP^2/\|\mathbf{x}-\hat{\mathbf{x}}\|^2)$, where $\hat{\mathbf{x}}$ denotes the estimated image and P represents the maximum possible intensity value in image $\mathbf{x}$.

Finally, our algorithms have been tested on real passive millimeter-wave (PMMW) images whose restoration constitutes another practical application of the proposed approaches. Recently, a series of PMMW imaging designs have been presented. Such imagers represent ideal candidates for the acquisition model in (2).

Fig. 1. For each image, 1^{st} column represents the original images; 2^{nd} column depicts the images of the 1^{st} column, degraded with a Gaussian PSF of variance 5; 3^{rd} column shows the blind restoration of the images through the first proposed method for SNR = 40 dB and CS ratio 30%.

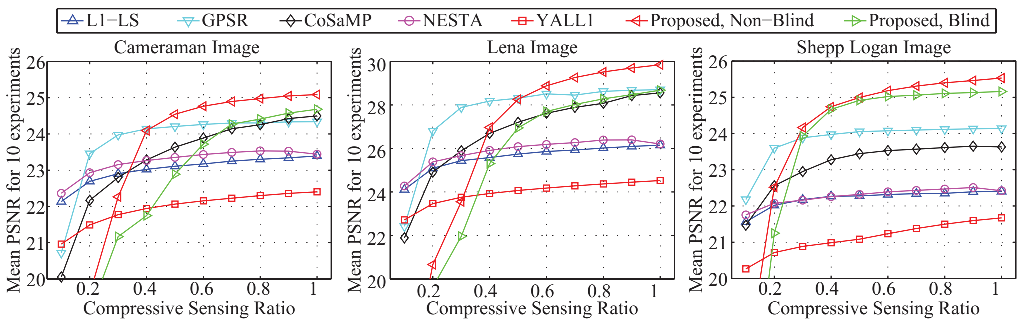

Fig. 2. Average performance comparison of the non-blind and blind versions of the second proposed approach with a series of algorithms, namely, *l1-ls*, *GPSR*, *NESTA*, *YALL1*, and *CoSaMP* that solve the non-blind reconstruction/restoration problem. All images are degraded with a Gaussian PSF of variance 5 and Gaussian noise is added to the CS measurements so that SNR = 40 dB.

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