Compressive sensing super resolution from multiple observations with application to passive millimeter wave images

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ABSTRACT

In this work we propose a novel framework to obtain high resolution images from compressed sensing imaging systems capturing multiple low resolution images of the same scene. The proposed approach of Compressed Sensing Super Resolution (CSSR), combines existing compressed sensing reconstruction algorithms with a low-resolution to high-resolution approach based on the use of a super Gaussian regularization term. The reconstruction alternates between compressed sensing reconstruction and super resolution reconstruction, including registration parameter estimation. The image estimation subproblem is solved using majorization-minimization while the compressed sensing reconstruction becomes an l1-minimization subject to a quadratic constraint. The performed experiments on grayscale and synthetically compressed real millimeter wave images, demonstrate the capability of the proposed framework to provide very good quality super resolved images from multiple low resolution compressed acquisitions.

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1. Introduction

Compressed Sensing (CS) theory offers a framework to simultaneously sense and compress signals. It establishes that a sparsely representable image/signal can be recovered from a highly incomplete set of measurements [1–3].

The design of CS image/video cameras (see [4–8]) has fostered the application of typical image processing tasks to CS observed images. CS has been applied to areas like radar analysis, face recognition, biomedical imaging, and microscopy imaging techniques [2, 9,10], among others. CS measurements have also been used to recover images observed through unknown blur [11,12].

Super Resolution (SR) from a single image has also benefited from the introduction of CS theory. In [13,14] learning based SR is used to estimate a High Resolution (HR) image from the CS observation of a downsampled remote sensing image. In [15] the downsampling is incorporated in the measurement matrix, the CS image is reconstructed in the wavelet domain and the signal is deconvolved in the Fourier domain.

The recovery of an HR image from a set of unregistered LR CS observations has been scarcely addressed in the literature. To the best of our knowledge, the only reported works treating this general SR problem are [16,17]. In these papers CS and LR to HR techniques are coupled, using a fast and simple registration method, which uses the reconstructed HR images instead of the LR ones [17]. A non-robust prior model on the original image to be reconstructed was used in both papers.

This paper also deals with the reconstruction of an HR image from a group of LR CS observed images. The proposed method assumes that the HR image to be estimated is compressible and, consequently, its warped, blurred, and downsampled versions are also compressible (see [11,12]). They can then be reconstructed from their CS observations. However, instead of first recovering the LR observations and then using LR to HR techniques we propose a combined framework where LR reconstructions and HR estimation are carried out simultaneously. This proposed method is based on a sound and well founded method to estimate registration parameters in LR to HR problems and the use of a new robust sparse promoting prior for the original image.

The proposed framework has been tested, in the experimental section, on CS grayscale and Passive Millimeter Wave (PMMW) images. Without using CS measurements, the improvement of Passive Millimeter Wave (PMMW) images to perform detection tasks has
been addressed in [18–22], and the use of CS techniques to reduce the time needed to capture such images has been addressed in [5,6,23,24]. In [17], high resolution images were super resolved for the first time, from multiple CS observations of unregistered LR PMMW images. We believe that PMMW images represent an important application area where CS and LR to HR techniques can be combined to enhance the detection capabilities of current PMMWI systems.

Before going into details, the more frequently used notation in the paper is listed next

\[
\begin{align*}
y_q &= M \times 1 \text{ compressed observation vector } q \in \{1, \ldots, Q\} \\
\Phi &= M \times D \text{ CS measurement matrix} \\
z_q &= D \times 1 \text{ the } q\text{-th LR image vector} \\
r_q &= D \times 1 \text{ CS observation noise vector} \\
A &= D \times N \text{ down-sampling matrix} \\
P &= \text{ zooming factor} \\
H_q &= N \times N \text{ blurring matrix} \\
C(s_q) &= N \times N \text{ warping matrix formed by motion vector } s_q \\
s_q &= 3 \times 1 \text{ motion vector (rotation } \theta_q, \text{ horizontal } c_q, \text{ and vertical } d_q \text{ displacements)} \\
x &= N \times 1 \text{ HR image vector} \\
w_q &= D \times 1 \text{ HR to LR acquisition noise vector} \\
B_q(s_q) &= D \times N \text{ LR acquisition model matrix} \\
a_q(s_q), b_q(s_q) &= N \times 1 \text{ pixel difference vectors} \\
D_{a_q}(s_q), b_q(s_q) &= N \times N \text{ diagonal matrix with } a_q(s_q) \text{ in the diagonal} \\
I &= \text{ the identity matrix} \\
L_{bl}(s_q) &= \text{ bottom-left-pixel matrix (br, tl, tr): (bottom-right, top-left, top-right)} \\
N_q &= M \times 1 \text{ combined CS and LR acquisition noise vector} \\
W &= D \times D \text{ transformation matrix} \\
a_q &= D \times 1 \text{ transformation coefficient vector} \\
\alpha, \beta, \eta, \tau &= \text{ non-negative parameters} \\
Q(x) &= \text{ regularization term} \\
\omega_q^2 &= \text{ filtered image applying } F_d \text{ on } x \text{ in } d\text{-direction} \\
\lambda_q &= D \times 1 \text{ Lagrangian multiplier vector} \\
\end{align*}
\]

The rest of this paper is organized as follows. The problem modeling and its formulation as an optimization task are presented in Sections 2 and 3, respectively. The estimation process is described in Section 4. We demonstrate the effectiveness of the proposed method in the experimental section, Section 5. Finally, conclusions are drawn in Section 6.

2. Modeling

In this work we assume that we have access to a set of Q CS LR observations of the form

\[
y_q = \Phi z_q + r_q \quad q = 1, \ldots, Q, 
\]

where \(y_q\) is an \(M \times 1\) vector representing compressed observations from the LR image \(z_q\), \(\Phi\) is a \(CS \times D\) measurement matrix, \(z_q\) is a column vector of size \(D \times 1\) representing the \(q\)-th LR image and \(r_q\) represents the observation noise. We denote by \(R\) the compression ratio of the measurement system, that is \(R = M/D, R \leq 1\). The sensing matrix \(\Phi\) consists of either real or binary entries. The matrix used in our work is binary, since it is easier to be synthesized physically [6,23,24]. In both cases the rows/columns of \(\Phi\) are normalized to 1. We assume that the LR observations \(z_q\) are related to an HR image of size \(N\), denoted by the column vector \(x\) by

\[
z_q = AH_q C(s_q)x + w_q = B_q(s_q)x + w_q, 
\]

where \(A\) is a \(D \times N\) down-sampling matrix, \(D \leq N\), which models the limited resolution of the acquisition system, when capturing the high resolution image, where \(N = P^2D\) and \(P \geq 1\) is the zooming factor, in each dimension of the image. \(H_q\) is an \(N \times N\) blurring matrix, modeling the action accompanying the imaging process. In this work, \(H_q\) is assumed to be known. \(C(s_q)\) is the \(N \times N\) warping matrix formed by motion vector \(s_q = [\theta_q, c_q, d_q]^T\), where \(\theta_q\) is the rotation angle, and \(c_q\) and \(d_q\) are respectively the horizontal and vertical translations of the \(q\)-th LR image with respect to the reference frame. Finally, \(w_q\) models the noise associated to the HR to LR acquisition process. We write \(B_q(s_q) = AH_q C(s_q)\) for simplicity.

As explained in [25], matrices \(C(s_q)\) can be explicitly stated as follows. Let us denote the coordinates of the reference HR grid by \((u, v)\) and the coordinates of the \(q\)-th warped HR grid, after applying \(C(s_q)\) to \(x\), by \((u_q, v_q)\). Then it holds that

\[
u_q = u \cos(\theta_q) - v \sin(\theta_q) + c_q \\
v_q = u \sin(\theta_q) + v \cos(\theta_q) + d_q. 
\]

Let us denote the displacements between the grids by \((u_q, v_q)^T = (u, v)^T - (u_q, v_q)^T\). The vector difference between the pixel at \((u_q, v_q)\) and the pixel at its top-left position in the reference HR grid is denoted by \((a_q(s_q), b_q(s_q))^T\) (see Fig. 1), that is,

\[
a_q(s_q) = \Delta u_q - \text{floor}(\Delta u_q), \\
b_q(s_q) = \Delta v_q - \text{floor}(\Delta v_q). 
\]

Using bilinear interpolation, the warped image \(C(s_q)x\) can be approximated as

\[
C(s_q)x \approx D_{a_q}(s_q)(I - D_{a_q}(s_q))L_{bl}(s_q)x + (I - D_{a_q}(s_q))D_{b_q}(s_q)L_{fr}(s_q)x \\
+ (I - D_{b_q}(s_q))L_{tl}(s_q)x \\
+ D_{b_q}(s_q)D_{a_q}(s_q)L_{br}(s_q)x, 
\]

where \(D_{a_q}(s_q)\) and \(D_{b_q}(s_q)\) denote diagonal matrices with the vectors \(a_q(s_q)\) and \(b_q(s_q)\) in their diagonals, respectively. \(I\) is the identity matrix. Matrices \(L_{x}\) with \(z \in \{bl(s_q), br(s_q), tl(s_q), tr(s_q)\}\) are constructed in such a way that the product \(L_{x}x\) produces pixels at the bottom-left, bottom-right, top-left, and top-right, locations of \((u_q, v_q)\), respectively.

Using (1) and (2) we can write

\[
y_q = \Phi B_q(s_q)x + n_q, \quad q = 1, \ldots, Q, 
\]

where \(n_q\) represents the combined CS and LR acquisition noise and \(x\) is the HR image we want to estimate.

3. Problem formulation

Since \(z_q\) in (2) represents translated and rotated LR versions of the original image \(x\) (which are assumed to be compressible in a transformed domain), we can estimate the original HR image by first recovering the LR images using CS techniques and then recover the HR image using standard super resolution techniques on the recovered low resolution images. To be precise, if we assume that the LR images are sparse in a transformed domain with \(z_q = W\hat{x}_q\), where \(W\) is a sparse promoting transformation of size \(D \times D\), we can recover them from the model in (1) by solving

\[
\hat{x}_q = \arg \min_{\hat{x}_q} \frac{\beta}{2} \| \Phi W\hat{x}_q - y_q \|^2 + \tau \| \lambda_q \|_1, 
\]

where \(\beta, \tau\) are regularization parameters, \(\| . \|\) is the Euclidean norm, and \(\| . \|_1\) the \(\ell_1\) norm. Then defining \(\hat{z}_q = W\hat{x}_q\) and \(s = (s_1, \ldots, s_Q)\) and using the degradation model (2), we can estimate the original image by solving

\[
\hat{x}, \hat{s} = \arg \min_{x, s} \frac{\beta}{2} \sum_q \| B_q(s_q)x - \hat{z}_q \|^2 + \alpha Q(x), 
\]

where \(\alpha\) is a regularization parameter, \(\| . \|\) is the Euclidean norm, and \(\| . \|_1\) the \(\ell_1\) norm.
where $\alpha$ and $\beta$ are non-negative parameters, and the remaining terms are described in details next. In [16,17] the following regularization term, $Q(x)$, was used for the image

$$Q(x) = \sum_{d \in \Delta} \sum_{i=1}^{N} \log(|ao^d(i)|),$$  

(11)

where $ao^d(i)$ is the $i$-th pixel of the filtered image, and

$$ao^d = F_d x,$$

(12)

where $F_d$ is a high-pass filter operator, and the index $d \in \Delta$ identifies one of the members of the used filter set. In this paper we have used a filter set with elements $\Delta = [h,v, hv, hh, vv]$, where $h, v$ represent the first order horizontal and vertical difference filters, $hv$ and $vh$ represent first order differences along diagonals, and $hh$ and $vv$ the horizontal and vertical second order differences. The regularization term favors sparsity of the high-pass filtered images $F_d x$, and corresponds to the Super-Gaussian log prior used in blind deconvolution [26].

Since the log function cannot be differentiated at zero, we consider in this work the following robust version of the log regularizer

$$\log_e(|s|) = \begin{cases} \log(|s|) & \text{for } |s| \geq \epsilon \\ \frac{s^2}{2\epsilon^2} - \frac{1}{2} - \log(\epsilon) & \text{for } 0 \leq |s| \leq \epsilon \end{cases}$$  

(13)

and replace $Q(x)$ in (11) by

$$Q(x) = \sum_{d \in \Delta} \sum_{i=1}^{N} \log_e(|ao^d(i)|),$$

(14)

where we have removed the dependency of $Q(\cdot)$ on $\epsilon$ for simplicity.

We have two approaches to solve the CSSR problem: the sequential approach and the alternate approach (see [27]). The latter alternates between compressive sensing reconstruction, registration parameter estimation, and recovering of the HR image. The former approach estimates the unknowns sequentially, one after the other, as follows. Firstly, the LR images are reconstructed using (9), then motion parameters and the HR image are estimated using (10).

As we will show in the experimental section, combining the two optimization problems (9), (10) above into a simultaneous one leads to improved performance, as this enables better exploitation of the compressibility of the LR observations using the additional information obtained from the estimated HR image. Hence, in the following, this alternate approach has been adopted. According to it, let $a = (a_1, \ldots, a_q)$ and define

$$L(x, a) = \frac{1}{2} \sum_{q=1}^{Q} \| \Phi_w a_q - y_q \|^2 + \alpha \sum_{q=1}^{Q} |a_q| + \beta \sum_{q=1}^{Q} \| B_q(s_q) x - W a_q \|^2.$$  

(15)

Then we approach the Compressed-Sensing Super Resolution (CSSR) problem by solving the following constrained optimization problem

$$\min_{L(x, a)}$$

s.t. $B_q(s_q)x = W a_q,$ for $q = 1, \ldots, Q.$

(16)

This is the approach we will describe and use in the following section. Notice that in (15) we could have also introduced a regularizer on the motion vector $s = (s_1, \ldots, s_Q)$. However, we have experimentally found that it is not necessary to use regularization on the motion vectors.

4. A super-resolution from compressed sensing algorithm

The constrained optimization problem in (16) is converted into an unconstrained optimization one, using the Alternate Direction Method of Multipliers (ADMM) [28,29]. We define the following augmented Lagrangian functional

$$L(x, a, s, \lambda) = L(x, a) + \sum_{q=1}^{Q} \lambda_q \langle B_q(s_q) x - W a_q \rangle$$

$$+ \frac{\beta}{2} \sum_{q=1}^{Q} \| B_q(s_q) x - W a_q \|^2,$$

(17)

where $L(x, a)$ has been defined in (15) for $q = 1, \ldots, Q$. $\lambda_q$ are $D \times 1$ Lagrange multiplier vectors with $\lambda = (\lambda_1, \ldots, \lambda_Q)$, and $\beta$ is a non-negative parameter. The ADMM leads to the following sequence of iterative unconstrained problems,

$$x^{k+1} = \arg \min_{x} L(x, a^k, s^k, \lambda^k),$$

(18)

$$a^{k+1} = \arg \min_{a} L(x^{k+1}, a, s^k, \lambda^k),$$

(19)

$$s^{k+1} = \arg \min_{s} L(x^{k+1}, a^{k+1}, s, \lambda^k),$$

(20)
\[ \lambda_{q}^{k+1} = \lambda_{q}^{k} - \beta [B_{q}(s_{q}^{k+1})x^{k+1} - W_{q}x_{q}^{k+1}] \quad q = 1, \ldots, Q, \tag{21} \]

where \( k \) is the iteration number. Notice that according to the ADMM approach, \( B_{q}(s_{q}) \) in (16) should not depend on the iteration index, as is the case here. However, we have not encountered any convergence issues with this iterative procedure.

Let us now describe the estimation process. The calculation of each \( \lambda_{q}^{k+1} \) is straightforward. The function \( \rho_{s}(s) = \log_{2}(s) \) in (14) is symmetric around 0, and \( \rho(\sqrt{s}) \) is concave and increasing for \( s \in [0, \infty) \) [26]. So, it can be represented as (see [30])

\[ \rho_{s}(s) = \inf_{\xi > 0} \left\{ \frac{1}{2} \xi s^{2} - \rho_{s}^{*}(\frac{1}{2} \xi) \right\}, \tag{22} \]

where \( \rho_{s}^{*}(\frac{1}{2} \xi) \) is the conjugate function

\[ \rho_{s}^{*}(\frac{1}{2} \xi) = \inf_{s > 0} \left\{ \frac{1}{2} s \xi^{2} - \rho_{s}(s) \right\}. \tag{23} \]

It is shown in [26] that the infimum in (22) is achieved when \( \xi = \rho_{s}^{*}(\xi) \). Consequently, for the regularization term \( Q(x) \) in (15), we can write

\[ Q(x) \leq R(x, \xi) = \frac{1}{2} \sum_{d \in \Delta} x^{T} F_{d} \Omega_{d} F_{d} x - \sum_{d \in \Delta} \sum_{i = 1}^{N} \rho_{s}^{*}(\frac{1}{2} \xi \eta(i)) \]

where \( \xi = (\xi_{1}, \ldots, \xi_{q}) \), \( \xi_{q} = (\xi_{q}(1), \ldots, \xi_{q}(N)) \) for \( q = 1, \ldots, Q \), with all its components positive, and \( \Omega_{d} \) is a diagonal matrix with entries

\[ \Omega_{d}(i, i) = \xi_{q}(i). \tag{25} \]

For a given \( x \), the first inequality in (24) becomes an equality if we have [26] for details.

\[ \xi_{q}(i) = \min(1/|\omega_{q}^{2}(i)|, 1/\epsilon^{2}) = \begin{cases} \frac{1}{|\omega_{q}^{2}(i)|} & \text{for } |\omega_{q}^{2}(i)| \geq \epsilon \\ \epsilon & \text{for } 0 \leq |\omega_{q}^{2}(i)| \leq \epsilon \end{cases} \tag{26} \]

where \( \omega_{q}^{2}(i) \) is defined from \( x \) in (12). Then we can apply standard Majorization-Minimization (MM) methods [31]. Given \( x^{k}, a^{k}, s^{k} \) and defining

\[ L^{k}(x) = \frac{\beta}{2} \sum_{q} \| B_{q}(s_{q}^{k})x - W_{q}x_{q}^{k} \|^{2} + \sum_{q} \lambda_{q}^{k} \| B_{q}(s_{q}^{k})x - W_{q}x_{q}^{k} \|^{2} \]

it can be easily shown that

\[ L^{k}(x^{k}) + \alpha Q(x^{k}) \geq L^{k}(x^{k+1}) + \alpha Q(x^{k+1}) \tag{28} \]

where

\[ x^{k+1} = \arg \min_{x} \left\{ \frac{\beta}{2} \sum_{q} \| B_{q}(s_{q}^{k})x - W_{q}x_{q}^{k} \|^{2} + \alpha R(x, \xi^{k}) + \sum_{q} \lambda_{q}^{k} \| B_{q}(s_{q}^{k})x - W_{q}x_{q}^{k} \|^{2} \right\}. \tag{29} \]

From (29), the optimization step in (18) produces the following linear equation for \( x^{k+1} \)

\[ x^{k+1} = \left[ \frac{\beta}{2} \sum_{d \in \Delta} B_{d}(s_{d}^{k})B_{d}^{T}(s_{d}^{k}) + \alpha \sum_{d \in \Delta} F_{d} \Omega_{d} F_{d} \right]^{-1} \times \sum_{q} B_{q}^{T}(s_{q}^{k}) \left[ \beta W_{q}x_{q}^{k} - \lambda_{q}^{k} \right] \tag{30} \]

\[ \Omega_{d}(i, i) = \min(1/|\omega_{q}^{2}(i)|, 1/\epsilon^{2}). \tag{31} \]

The optimization step in (19) for each \( a_{q} \) produces

\[ a_{q}^{k+1} = \arg \min_{a_{q}} \left\{ \frac{\eta}{2} \| \Phi W_{q} - y_{q} \|^{2} + \tau \| a_{q} \|_{1} ight\} \tag{32} \]

which is equivalent to

\[ a_{q}^{k+1} = \arg \min_{a_{q}} \left\{ \frac{\eta}{2} \| \Phi W_{q} - y_{q} \|^{2} + \tau \| a_{q} \|_{1} \right\}. \tag{33} \]

The above equation can be rewritten as

\[ a_{q}^{k+1} = \arg \min_{a_{q}} \| \Phi' W_{q} - J' \|^{2} + \tau \| a_{q} \|_{1}. \tag{34} \]

where

\[ J' = \left[ \sqrt{\frac{2}{\beta}} \right] \] and \( \Phi' = \left[ \frac{\sqrt{2}}{\beta} \right] \]

with \( I \) the \( D \times D \) identity matrix.

The above optimization problem can be solved using the algorithm in [32].

To estimate the registration parameters in (20), we have to solve

\[ s_{q}^{k+1} = \arg \min_{s_{q}} \frac{\beta}{2} \| B_{q}(s_{q})x^{k+1} - W_{q}x_{q}^{k+1} \|^{2}. \tag{36} \]

Notice that we could use regularization of the parameters to be estimated as we did in [17]. However, we have experimentally observed that regularization was not needed for this problem. \( B_{q}(s_{q})x \) can be approximated by expanding it into its first-order Taylor series around the previous value \( s_{q}^{k} \). Hence obtaining [see [33, 35]]

\[ B(s_{q}^{k})x^{k+1} \approx B(s_{q}^{k})x^{k+1} \]

\[ + \left[ O_{q1}(s_{q}^{k})x^{k+1}, O_{q2}(s_{q}^{k})x^{k+1}, O_{q3}(s_{q}^{k})x^{k+1} \right] \]

\[ \times (s_{q}^{k} - s_{q}) \]

where \( O_{q1}(s_{q}^{k})x^{k+1} \approx AH_{q}N_{q}(s_{q}^{k})x^{k+1} \). The values of \( N_{q}(s_{q})x^{k+1} \) can be calculated using

\[ N_{q}(s_{q}^{k})x = N_{2}(s_{q}^{k})x, N_{3}(s_{q}^{k})x \]

\[ = \left[ P_{1}(s_{q}^{k})M_{1}(s_{q}^{k}) + P_{2}(s_{q}^{k})M_{2}(s_{q}^{k}), M_{1}(s_{q}^{k}), M_{2}(s_{q}^{k}) \right]. \tag{38} \]

where

\[ M_{1}(s_{q}^{k}) = (I - D_{b}(s_{q}^{k}))(L_{b}(s_{q}^{k}) - L_{b}(s_{q}^{k})), D_{b}(s_{q}^{k}))(L_{b}(s_{q}^{k}) - L_{b}(s_{q}^{k})) \]

\[ M_{2}(s_{q}^{k}) = (I - D_{b}(s_{q}^{k}))(L_{b}(s_{q}^{k}) - L_{b}(s_{q}^{k})), D_{b}(s_{q}^{k}))(L_{b}(s_{q}^{k}) - L_{b}(s_{q}^{k})) \]

\[ P_{1}(s_{q}^{k}) = [Du \sin(\theta_{q}^{k}) + Du \cos(\theta_{q}^{k})] \]

\[ P_{2}(s_{q}^{k}) = [Du \cos(\theta_{q}^{k}) - Du \sin(\theta_{q}^{k})] \]

and \( Du \) and \( Dv \) are diagonal matrices whose diagonals are the vectors \( u \) and \( v \), respectively. Substituting (37) into (36), we obtain the final update equation as follows
Algorithm 1 Compressive Sensing Super Resolution (CSSR).

Require: Values α, β, τ, η
Initialize $a^0, s^0, \lambda^0$, $\Omega^q = \{s^0_d, d \in \Delta\}$

while convergence criterion is not met do
1. Calculate $x^{k+1}$ by solving (30)
2. For $d \in \Delta$, calculate $\Omega^{q+1}_d$ using (31)
3. For $q = 1, \ldots, Q$, calculate $a^{k+1}_q$ using (34)
4. For $q = 1, \ldots, Q$, calculate $s^{k+1}_q$ using (43)
5. For $q = 1, \ldots, Q$, update $\lambda^{q+1}_q$ using (21)
6. Set $k = k + 1$
return $x$

$$s^{k+1}_q = \left[A^k_q\right]^{-1} (Y^k_q + A^k_q s^k_q) = s^k_q + \left[A^k_q\right]^{-1} Y^k_q,$$ (43)

where $A^k_q$ and $Y^k_q$ correspond to the $q$-th observation at the $k$-th iteration. The $i, j \in \{1, 2, 3\}$ elements of these matrices are given by

$$A^k_{q ij} = \left[\Phi^k q_i, \Phi^k q_j\right] x^{k-1} + \left[\Phi^k q_i, \Phi^k q_j\right] x^{k-2} + \ldots + \left[\Phi^k q_i, \Phi^k q_j\right] x^0 + \left[\Phi^k q_i, \Phi^k q_j\right] x^1,$$ (44)

The complete CSSR algorithm is presented in Algorithm 1.

5. Experimental results

To evaluate the proposed algorithm, experiments were carried out on two types of images: simulated compressed LR images, and real LR images which are synthetically compressed using a measurement matrix $\Phi$. In both cases, $\Phi$ is constructed using a circulant Toeplitz matrix with entries drawn from a Bernoulli distribution. We utilized a 3-level Haar wavelet transform as the transform basis. Using exhaustive search, the values of the parameters utilized were $\tau = 1.25 \times 10^{-3}, \alpha = 5 \times 10^{-3}$, for all experiments. For the SNR = 40 dB case, $\eta = 49.29, \beta = 3.85 \times 10^{-3}$, and for the SNR = 30 dB case, $\eta = 4.93, \beta = 3.85 \times 10^{-4}$.

The algorithm stops when either a maximum number of iterations (60 in all experiments) is reached, or when

$$\frac{\|x^k - \hat{x}^{k-1}\|}{\|x^k - \hat{x}^{k-2}\|} \leq 10^{-4}.$$ (45)

To estimate the initial value of the registration parameters $s^0_q$ in (43), the following minimization problem, similar to (36), was solved

$$s^0_q = \text{argmin}_{s_q} \| \mathbf{C}(s_q) \mathbf{W}a^0_q - \mathbf{W}a^0_q \|^2, \quad q = 2, \ldots, Q$$ (46)

The obtained registration parameters are then upscaled to the HR level. The subscript $R$ denotes the reference image which corresponds to the first reconstructed LR observation.

5.1. Simulated images

To generate all simulated LR images from an original image we use the following procedure. The original HR image is first randomly rotated and horizontally and vertically displaced. Then it is blurred with a Gaussian blur, with known variance. Next it is downsampled by a variable zooming factor $P$. It is compressed using $\Phi$, with variable compression ratio $R$, and finally white Gaussian noise is added to the CS observations with variable signal-to-noise ratios (SNR). $Q$ different observations are generated using this procedure. An example of the degradation process, on the Cameraman image, is shown in Fig. 2.

As performance measure we use the Peak Signal to Noise Ratio (PSNR) calculated using

$$\text{PSNR} = 10 \log \frac{N_{\text{max}}(x_{\text{orig}})}{\| x_{\text{est}} - x_{\text{orig}} \|^2},$$ (47)

where $x_{\text{orig}}$ and $x_{\text{est}}$ are $N \times 1$ vectors, representing the original and the estimated HR image, respectively, and $max(x_{\text{orig}})$ is the maximum possible value of the image $x_{\text{orig}}$ (the value is equal to 255 for an 8-bit image).

5.1.1. Compressed sensing reconstruction

This experiment concentrates on the CS reconstruction process, to study the performance of the CSSR algorithm. We use two simulated LR images as follows. With the HR Shepp–Logan image of size 256 × 256 pixels, shown in Fig. 3(a), we used a blur with variance $= 3$, zooming factor $P = 2$, and warping matrix corresponding respectively to the following rotation and displacement vectors: $[0.0, 0]^T, [-0.047, 2, -3]^T$. The two simulated LR images are shown in Fig. 3(b, c). These simulated LR images are then compressed with compression ratio $R = 0.8$, and noise is added with SNR = 30 dB.

Let us examine the first iteration resulting LR estimations using (33), without the registration regularization term. The estimated
images are shown in Fig. 3(d, e), and the PSNR values of the reconstructed LR images are 40.22 dB and 40.16 dB, respectively. As the CSR process advances, the inclusion of the registration term allows for a better extraction of the information in the compressed observations; the final estimated LR images are shown in Fig. 3(f, g), with PSNR values being 42.88 dB and 42.77 dB, respectively.

For the Cameraman image shown in Fig. 4(a), the downsampling images are shown in Fig. 4(b, c), the initially estimated LR images are shown in Fig. 4(d, e), with PSNR = 35.52 dB and 35.29 dB, respectively. The final estimated LR images are shown in Fig. 4(f, g), with PSNRs equal to 35.55 dB and 35.34 dB, respectively.

For the Lena image shown in Fig. 5(a), the downsampling images are shown in Fig. 5(b, c), the initially estimated LR images are shown in Fig. 5(d, e), with PSNR = 32.45 dB and 32.28 dB, respectively. The final estimated LR images are shown in Fig. 5(f, g), with PSNRs equal to 34.24 dB and 34.07 dB, respectively.

Notice that the inclusion of the registration regularization term in (33) greatly contributes to the PSNR improvement when using the CSR algorithm. This improvement will be very useful when super resolving the LR observations to estimate the HR image.

### 5.1.4. The general case

In this experiment the overall behavior of the CSR algorithm is investigated; the compression ratios and zooming factors will be varied and the sequential and alternate approaches compared. A comparison between the sequential approach and the CSR algorithm on the Cameraman and Shepp–Logan images is shown in Tables 3 and 4, respectively. Fig. 7 shows for $P = 2$ and $4$ with $Q = 4$, Blur variance $3$, SNR = 40 dB, $R = 0.8$, the estimated Shepp–Logan images.

The obtained results show how the alternate approach outperform the sequential. It is important to pause to examine the obtained results. For a $256 \times 256$ HR image, the total number of pixels is 65536. If we use a zooming factor $P = 4$, then the size of the LR image will be $64 \times 64$, and the total number of pixels 4096. Using a compression ratio $R = 0.8$, the number of projections is 3277. Instead of saving 65536 pixel values, the CSR algorithm uses only 3277 values, a percentage of $4 \times 5.0\%$, still adequate to obtain a good HR image, see Fig. 7(b).

Let us now examine the behavior of the CSR algorithm as a function of several factors. For $Q = 4$, $P = 4$, SNR = 30 dB, Fig. 8

### Table 1

<table>
<thead>
<tr>
<th>Zooming factor</th>
<th>Motion vector</th>
<th>$a_l$</th>
<th>$c_l$</th>
<th>$d_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observation, $q = 2$</td>
<td>True</td>
<td>0.5236</td>
<td>2.99</td>
<td>-3.00</td>
</tr>
<tr>
<td></td>
<td>Estimated</td>
<td>0.5229</td>
<td>1.999</td>
<td>-3.002</td>
</tr>
<tr>
<td></td>
<td>Abs. error</td>
<td>7.0e-5</td>
<td>1.0e-3</td>
<td>2.0e-3</td>
</tr>
<tr>
<td>Observation, $q = 3$</td>
<td>True</td>
<td>-0.06916</td>
<td>-1.002</td>
<td>-2.005</td>
</tr>
<tr>
<td></td>
<td>Estimated</td>
<td>-0.06902</td>
<td>-1.012</td>
<td>-2.046</td>
</tr>
<tr>
<td></td>
<td>Abs. error</td>
<td>7.9e-4</td>
<td>1.2e-2</td>
<td>4.6e-2</td>
</tr>
<tr>
<td>Observation, $q = 4$</td>
<td>True</td>
<td>-0.03491</td>
<td>3.0</td>
<td>-1.0</td>
</tr>
<tr>
<td></td>
<td>Estimated</td>
<td>-0.03478</td>
<td>3.001</td>
<td>-1.001</td>
</tr>
<tr>
<td></td>
<td>Abs. error</td>
<td>1.3e-4</td>
<td>1.0e-3</td>
<td>1.0e-3</td>
</tr>
<tr>
<td></td>
<td>Estimated</td>
<td>-0.03351</td>
<td>2.999</td>
<td>-1.041</td>
</tr>
<tr>
<td></td>
<td>Abs. error</td>
<td>1.4e-3</td>
<td>1.0e-3</td>
<td>4.1e-2</td>
</tr>
</tbody>
</table>
shows the behavior of CSSR when the compression ratio and blur vary.

Table 2
Comparison of state-of-the-art SR algorithms with the CSSR algorithm, with \( Q = 4 \) and for CSSR \( R = 1.0 \). Every experiment was repeated three times and the shown values are the average values. In bold are the highest PSNR values.

<table>
<thead>
<tr>
<th>Blur Var</th>
<th>3</th>
<th>5</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR 40</td>
<td>22.5</td>
<td>22.7</td>
<td>22.4</td>
</tr>
<tr>
<td>30</td>
<td>20.2</td>
<td>20.3</td>
<td>20.2</td>
</tr>
<tr>
<td>40</td>
<td>21.0</td>
<td>21.1</td>
<td>21.0</td>
</tr>
<tr>
<td>5</td>
<td>22.2</td>
<td>22.3</td>
<td>22.2</td>
</tr>
<tr>
<td>9</td>
<td>21.6</td>
<td>21.7</td>
<td>21.6</td>
</tr>
</tbody>
</table>

Alg P Average PSNR (dB)

Image Cameraman

| BIC | 2 | 22.5 | 22.7 | 22.4 | 22.3 | 22.4 | 22.4 |
| L1S | 2 | 24.1 | 24.5 | 23.9 | 24.2 | 24.1 | 23.9 |
| SnS | 2 | 24.1 | 23.1 | 24.1 | 23.4 | 24.2 | 23.3 |
| FRSR | 2 | 20.9 | 21.5 | 19.3 | 19.8 | 21.5 | 20.6 |
| RSR | 2 | 22.2 | 22.4 | 22.1 | 22.8 | 22.2 | 22.1 |
| CSSR | 2 | 25.9 | 24.8 | 24.6 | 23.8 | 24.4 | 22.6 |
| 4 | 25.0 | 24.1 | 24.0 | 23.3 | 23.1 | 22.3 |

Image Lena

| BIC | 2 | 24.7 | 24.7 | 24.6 | 24.6 | 24.5 | 24.4 |
| L1S | 2 | 27.0 | 27.4 | 27.2 | 27.5 | 26.7 | 27.4 |
| SnS | 2 | 27.7 | 27.5 | 28.4 | 27.6 | 28.3 | 27.5 |
| FRSR | 2 | 20.5 | 20.8 | 21.2 | 20.9 | 22.0 | 20.9 |
| RSR | 2 | 23.1 | 25.2 | 25.0 | 25.4 | 24.2 | 23.0 |
| CSSR | 2 | 25.1 | 29.0 | 29.4 | 27.6 | 27.3 | 26.0 |
| 4 | 29.4 | 27.4 | 28.2 | 26.4 | 26.7 | 25.2 |

Image Shepp–Logan

| BIC | 2 | 20.5 | 20.5 | 20.5 | 20.8 | 20.3 | 20.3 |
| L1S | 2 | 25.7 | 27.9 | 27.9 | 30.3 | 26.2 | 27.7 |
| SnS | 2 | 24.1 | 23.3 | 24.1 | 23.2 | 24.1 | 23.2 |
| FRSR | 2 | 20.0 | 20.8 | 20.2 | 21.9 | 20.2 | 21.0 |
| RSR | 2 | 21.1 | 21.0 | 21.4 | 21.0 | 21.3 | 21.1 |
| CSSR | 2 | 25.8 | 26.4 | 24.4 | 24.9 | 22.9 | 23.2 |
| 4 | 24.7 | 25.3 | 23.7 | 24.1 | 22.4 | 22.6 |

Fig. 6. Comparison among SR algorithms and the CSSR algorithm, \( P = 4 \), \( Q = 4 \), and for CSR, \( R = 1.0 \). (a) Cameraman image, (b) Lena, (c) Shepp–Logan image.

Fig. 7. Image super resolution from simulated images, \( Q = 4 \), Blur Var = 3, SNR = 40 dB, (a) Estimated image using \( P = 2 \), R = 0.8, (b) Estimated image using \( P = 4 \), \( R = 0.8 \).

Fig. 10 shows the results obtained for both images, with \( Q = 4 \), Blur Var = 5, \( P = 2 \). In general, and as expected, better PSNRs were obtained with less noise and less compression. Notice, however, the interesting behavior of the test on the Shepp–Logan image where better quality restored images are obtained from noisier observations for low compression ratios. This is very likely due to the very flat nature of this particular image.

5.2. Real images

This experiment uses real LR observations, the underlying HR image being unavailable. The experiments were carried out to vi-
Table 3
Comparison of the sequential approach and the CSSR algorithm for the Cameraman image, using Q = 4. Every experiment was repeated three times and the shown values are the average values. In bold are the highest PSNR values.

<table>
<thead>
<tr>
<th>Blur Var</th>
<th>3</th>
<th>5</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>40</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>P</td>
<td>Alg</td>
<td>R</td>
<td>Average PSNR (dB)</td>
</tr>
<tr>
<td>Sequential</td>
<td>0.2</td>
<td>19.4</td>
<td>19.9</td>
</tr>
<tr>
<td>0.4</td>
<td>22.2</td>
<td>22.1</td>
<td>21.7</td>
</tr>
<tr>
<td>0.6</td>
<td>23.8</td>
<td>22.9</td>
<td>23.0</td>
</tr>
<tr>
<td>0.8</td>
<td>24.5</td>
<td>23.2</td>
<td>23.5</td>
</tr>
<tr>
<td>1.0</td>
<td>24.8</td>
<td>23.4</td>
<td>23.8</td>
</tr>
<tr>
<td>CSSR</td>
<td>0.2</td>
<td>22.2</td>
<td>20.7</td>
</tr>
<tr>
<td>0.4</td>
<td>24.2</td>
<td>22.7</td>
<td>23.4</td>
</tr>
<tr>
<td>0.6</td>
<td>25.1</td>
<td>23.8</td>
<td>24.0</td>
</tr>
<tr>
<td>0.8</td>
<td>25.7</td>
<td>24.4</td>
<td>24.4</td>
</tr>
<tr>
<td>1.0</td>
<td>25.9</td>
<td>24.8</td>
<td>24.6</td>
</tr>
</tbody>
</table>

Table 4
Performance of sequential approach, with CSSR algorithm, for the Shepp-Logan image, using Q = 4. Every experiment was repeated three times and the shown values are the average values. In bold are the highest PSNR values.

<table>
<thead>
<tr>
<th>Blur Var</th>
<th>3</th>
<th>5</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR</td>
<td>40</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>P</td>
<td>Alg</td>
<td>R</td>
<td>Average PSNR (dB)</td>
</tr>
<tr>
<td>Sequential</td>
<td>0.2</td>
<td>17.1</td>
<td>18.6</td>
</tr>
<tr>
<td>0.4</td>
<td>22.8</td>
<td>23.3</td>
<td>21.9</td>
</tr>
<tr>
<td>0.6</td>
<td>24.3</td>
<td>23.9</td>
<td>23.1</td>
</tr>
<tr>
<td>0.8</td>
<td>24.7</td>
<td>24.1</td>
<td>23.5</td>
</tr>
<tr>
<td>1.0</td>
<td>24.9</td>
<td>24.1</td>
<td>23.7</td>
</tr>
<tr>
<td>CSSR</td>
<td>0.2</td>
<td>23.3</td>
<td>21.7</td>
</tr>
<tr>
<td>0.4</td>
<td>25.0</td>
<td>24.3</td>
<td>23.8</td>
</tr>
<tr>
<td>0.6</td>
<td>25.5</td>
<td>25.7</td>
<td>24.3</td>
</tr>
<tr>
<td>0.8</td>
<td>25.7</td>
<td>26.2</td>
<td>24.4</td>
</tr>
<tr>
<td>1.0</td>
<td>25.8</td>
<td>26.4</td>
<td>24.4</td>
</tr>
</tbody>
</table>

In the first experiment we used noisy LR images of a car, the first four of which are shown in Fig. 11(a). The estimated high resolution image, using bilinear interpolation of one of the LR reconstructed images, is shown in Fig. 11(b). The estimated HR images using the CSSR algorithm are shown in Figs. 11(c, d), for number of input images Q = 4 and 16, respectively. We used a compression ratio R = 0.8, and a zooming factor, P = 2. The better performance of the proposed CSSR in comparison to bilinear interpolation is clearly observable. Notice also that a better performance is achieved by increasing the number of input images; this becomes apparent when trying to read the letters written on the windscreen of the car; they are more readable in Fig. 11(d) than that in Fig. 11(c). Notice also the artifacts on the left hand side of the estimated image, due to warping and Fourier implementation of the blur convolution.

In the second experiment we used noisy LR images of a toy, the first four of which are shown in Fig. 12(a). The bilinear interpolation of one reconstructed LR image is shown in Fig. 12(b). The estimated images using the CSSR algorithm, are shown in Figs. 12(c, d), for number of input images Q = 4 and 16, respec-
tively. The compression ratio was equal to 0.8, and the zooming factor was equal to 2. Similarly, the best result was obtained using more input images (as shown in Fig. 12(d)), when compared to both the interpolated image in Fig. 12(b), and to the CSSR estimated image in Fig. 12(c). The estimated image in Fig. 12(d) is smoother with sharper edges and better quality.

5.3. Real passive millimeter wave images

In this experiment we used four noisy real 100 × 30 PMMW LR observations of a man hiding a threat attached to his left arm, as shown in Fig. 13(a). The unregistered images were synthetically compressed using a compression ratio R = 0.8. In the first part we used a zooming factor P = 2, and compare the estimated image using CSSR, shown in Fig. 13(c) with the image shown in Fig. 13(b), obtained by bilinear interpolation of the reconstructed reference image. In the estimated image using CSSR, the discontinuity in the arm, which refers to the threat location, appears more prominent than that in the interpolated image. This becomes even clearer in the next experiment, using a zooming factor P = 4, under the same conditions. The estimated image using CSSR, shown in Fig. 13(e), is again compared with the interpolated image, shown in Fig. 13(d). In addition, the edges of the legs look sharper. Notice that the estimated image in the latter case has achieved a factor of increase in resolution of 16, a good result taking into account the low quality of the compressed input images. This is a very important preprocessing step which should help improve the performance of any threat detection algorithm.

6. Conclusions

In this work we have proposed a method to obtain an HR image from a set of LR CS observations. The method combines CS and LR to HR reconstruction using ADMM and base the HR robust reconstruction on an efficient registration procedure and a new sparsity promoting prior. We have experimentally shown that this simultaneous reconstruction outperforms the method, that first performs LR reconstruction to then obtain an HR image from a set of LR observations. The experiments carried out show a very good performance in comparison with existing HR methods which do not use CS observations. We have also shown how the proposed method can be used to improve the quality of PMMW images to be used in detection tasks.

Acknowledgments

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![Fig. 10. Performance of the proposed CSSR for various signal to noise ratios, of the additive noise, and compression ratios. Q = 4, P = 2, BLR Var = 5, SNR = 30 dB. (a) Cameraman image, (b) Shepp–Logan image.](image)

![Fig. 11. Image super resolution from real observations, R = 0.8, P = 2. (a) First 4 LR images, (b) Bilinear interpolation of one reconstructed LR image, (c) Estimated HR image using the CSSR algorithm, with Q = 4, (d) Estimated HR image using the CSSR algorithm, with Q = 16.](image)

![Fig. 12. Image super resolution from real observations. R = 0.8, P = 2. (a) First 4 LR images, (b) Bilinear interpolation of one reconstructed LR image, (c) Estimated HR image using the CSSR algorithm, with Q = 4, (d) Estimated HR image using the CSSR algorithm, with Q = 16.](image)
Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.dsp.2015.12.005.

References

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Prof. Molina serves as an Associate Editor of Applied Signal Processing (2005–2007); the IEEE Transactions on Image Processing (2010–present); and Progress in Artificial Intelligence (2011–present); and an Area Editor of Digital Signal Processing (2011–present). He is the recipient of an IEEE International Conference on Image Processing Paper Award (2007) and an ISPA Best Paper Award (2009). He is a coauthor of a paper awarded the runner-up prize at Reception for early-stage researchers at the House of Commons.

Angelos K. Katsaggelos (Fellow, IEEE) received the Diploma degree in electrical and mechanical engineering from the Aristotelian University of Thessaloniki, Thessaloniki, Greece, in 1979 and the M.S. and Ph.D. degrees in electrical engineering from the Georgia Institute of Technology, Atlanta, GA, USA, in 1981 and 1985, respectively. In 1985, he joined the Department of Electrical Engineering and Computer Science at Northwestern University, Evanston, IL, USA, where he is currently a Professor holder of the AT&T chair. He was previously the holder of the Ameritech Chair of Information Technology (1997–2003). He is also the Director of the Motorola Center for Seamless Communications, a member of the Academic Staff, NorthShore University Health System, an affiliated faculty at the Department of Linguistics and he has an appointment with the Argonne National Laboratory. He has published extensively in the areas of multimedia signal processing and communications (over 230 journal papers, 500 conference papers, and 40 book chapters), and he is the holder of 25 international patents. He is the coauthor of Rate-Distortion Based Video Compression (Norwell, MA, USA: Kluwer, 1997), Super-Resolution for Images and Video (San Rafael, CA, USA: Claypool, 2007), Joint Source-Channel Video Transmission (San Rafael, CA, USA: Claypool, 2007), and Machine Learning, Optimization, and Sparsity (Cambridge, U.K.: Cambridge Univ. Press, forthcoming). He has supervised 50 Ph.D. dissertations so far.