USING LOGARITHMIC OPINION POOLING TECHNIQUES IN BAYESIAN BLIND MULTI-CHANNEL RESTORATION

Preparation of Camera-Ready Contributions to INSTICC Proceedings

Bruno Amizic, Aggelos K. Katsaggelos

Department of Electrical Engineering and Computer Sciance, Northwestern University, 2145 Sheridan Rd, Evanston, USA

bam660@eecs.northwestern.edu, aggk@eecs.northwestern.edu

Rafael Molina

Departamento de Ciencias de la Computación e I.A., Universidad de Granada, 18071 Granada, Spain

rms@decsai.ugr.es

Keywords: Bayesian framework, blind multi-channel restoration, logarithmic opinion pooling, variational methods.

Abstract: In this paper we examine the use of logarithmic opinion pooling techniques to combine two observations models that are normally used in multi-channel image restoration techniques. The combined observation model is used together with simultaneous autoregression prior models for the image and blurs to define the joint distribution of image, blurs and observations. Assuming that all the unknown parameters are previously estimated we use variational techniques to approximate the posterior distribution of the real underlying image and the unknown blurs. We will examine the use of two approximations of the posterior distribution. Experimental results are used to validate the proposed approach.

1 INTRODUCTION

Blind image restoration (BIR) has been an active research topic for many years now (for the recent literature review see (Bishop et al., 2007)). In the BIR solutions (Molina et al., 2006) both the original image and blur are considered to be unknown. Blind multi-channel restoration (BMCR) is an extension to the BIR problem when multiple views of the scene are available. Both BIR and BMCR are ill-possed problems. There are numerous practical applications in which BMCR can be used. Satellite imaging, remote sensing, astronomical imaging, microscopy and video processing are some of the applications where multiple distorted views of the original scene are available.

In this paper we propose solutions to the BMCR problem based on the Bayesian paradigm, which has already been widely used for image restoration (Molina et al., 1999), (Mateos et al., 2000), (Galatsanos et al., 2002), removal of blocking artifacts (Mateos et al., 2000) and deconvolution with partially known blurs (Galatsanos et al., 2002).

For our BMCR problem formulation it is assumed that L distorted versions of the original scene are available. Each observation is modeled by a Linear Space Invariant (LSI) system. Therefore, the output g_i for each individual channel is given in vector-matrix form by

$$\mathrm{g}_i = \mathrm{H}_i\mathrm{f} + \mathrm{n}_i, \;\; i = 1, 2, ..., \mathrm{L}, \tag{1}$$

where f is the original image, n_i represents the additive noise per channel, and H_i represents the unknown blur matrix which is approximated by an $\mathrm{N}\times\mathrm{N}$ block-circulant matrix (all vectors are of size $\mathrm{N}\times 1$). It should be pointed out that for the observations that are not spatially aligned, matrix H_i will also be used to model any possible spatial shift. Before we proceed further, let us rewrite Equation 1 in a more compact form as

$$\mathrm{g} = \mathrm{H}\mathrm{f} + \mathrm{n} = \mathrm{F}\mathrm{h} + \mathrm{n}, \tag{2}$$

where $\mathrm{g} = \left[\mathrm{g}_1^{\mathrm{T}}, \mathrm{g}_2^{\mathrm{T}}, ..., \mathrm{g}_{\mathrm{L}}^{\mathrm{T}}\right]^{\mathrm{T}}$, $\mathrm{h} = \left[\mathrm{h}_1^{\mathrm{T}}, \mathrm{h}_2^{\mathrm{T}}, ..., \mathrm{h}_{\mathrm{L}}^{\mathrm{T}}\right]^{\mathrm{T}}$, $\mathrm{H} = \left[\mathrm{H}_1^{\mathrm{T}}, \mathrm{H}_2^{\mathrm{T}}, ..., \mathrm{H}_{\mathrm{L}}^{\mathrm{T}}\right]^{\mathrm{T}}$, and $\mathrm{n} = \left[\mathrm{n}_1^{\mathrm{T}}, \mathrm{n}_2^{\mathrm{T}}, ..., \mathrm{n}_{\mathrm{L}}^{\mathrm{T}}\right]^{\mathrm{T}}$. Matrix F has size $\mathrm{LN}\times\mathrm{N}$ and represents the block diagonal convolutional matrix.

In this work, our goal is to formulate the BMCR problem by constructively combining different observations models and to use the variational approach to approximate the joint posterior distribution of the original image and blurs given the multi-channel observations.

This paper is organized as follows. In Section 2, we examine the Bayesian modeling of the multi-channel restoration problem which allows us to combine different observation models. In Section 3 we use variational techniques to approximate the joint posterior distribution of the original image and blurs given the multi-channel observations. In Section 4 we present experimental results and present the conclusions in Section 5.

2 BAYESIAN FRAMEWORK

On the unknown image we assume that its luminosity distribution is smooth, and therefore we choose the simultaneous autoregression (SAR) model (Ripley, 1981) as the image prior,

$$p(\mathrm{f} \mid \alpha_{im}) \propto \exp\left\{-\frac{1}{2}\alpha_{im}\|\mathrm{Cf}\|^2\right\}. \qquad (3)$$

Matrix C has size $\mathrm{N}\times\mathrm{N}$ and denotes the Laplacian operator, N is number of pixels in the image support and α_{im}^{-1} is the variance of the Gaussian distribution.

For the joint blur prior, it is assumed that the point spread function (PSF) of each individual channel independently follows the SAR model described by Equation 3, that is

$$p(\mathrm{h} \mid \alpha_{bl}) \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{\mathrm{L}}\alpha_{bl,i}\|\mathrm{Ch}_i\|^2\right\}, \qquad (4)$$

where $\alpha_{bl,i}^{-1}$ is the variance of the i^{th} channel blur and α_{bl} denotes set, $\{\alpha_{bl,i}\} : i = 1,2,..\mathrm{L}$.

From the observation model described in Equation 1 we obtain

$$p_1(\mathrm{g} \mid \mathrm{h},\mathrm{f},\beta) \propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{\mathrm{L}}\beta_i\|\mathrm{g}_i-\mathrm{H}_i\mathrm{f}\|^2\right\}, \qquad (5)$$

where β_i^{-1} is the variance of the i^{th} Gaussian noise vector and β denotes set, $\{\beta_i\} : i = 1,2,...,\mathrm{L}$.

We now want to introduce additional constraints on the blurring functions. Let us first assume that there is no noise in the observation process. Then we have

$$\mathrm{R_g} = \mathrm{HR_fH^T} \qquad (6)$$

where R denotes the autocorrelation matrix.

Now, if $\mathrm{R_f}$ has full rank; for any vector u we have

$$\mathrm{R_g u} = 0 \Longrightarrow \mathrm{H^T u} = 0, \qquad (7)$$

since if $\mathrm{H^T u} \neq 0$ then, for $\mathrm{R_f}$ being full rank, $\mathrm{u^T HR_f H^T u} \neq 0$ and so $\mathrm{R_g u} \neq 0$.

Furthermore, if additionally H has full column rank, N, then the rank of R_g is also N.

Consequently, the eigenvectors associated with the N largest eigenvalues of R_g span the signal subspace, whereas the eigenvectors associated with the $(\mathrm{L}-1)\mathrm{N}$ smallest eigenvalues span its orthogonal complement, the noise subspace. The signal subspace is also the subspace spanned by the columns of the filtering matrix H.

Let us denote each of the eigenvectors spanning the noise subspace by $\mathrm{u}_i,\ i = 1,\dots,(\mathrm{L}-1)\mathrm{N}$. Based on our previous assumptions about $\mathrm{R_g}$ and H we conclude that

$$\mathrm{H^T u}_i = 0, \qquad i = 1,\dots,(\mathrm{L}-1)\mathrm{N}. \qquad (8)$$

Then, the above equation can also be written as

$$\mathrm{V}_i\mathrm{h} = 0, \qquad i = 1,\dots,(\mathrm{L}-1)\mathrm{N}. \qquad (9)$$

where $\mathrm{V}_i = [\mathrm{V}_i^1, \mathrm{V}_i^2, ..., \mathrm{V}_i^{\mathrm{L}}]$ is an $\mathrm{N}\times\mathrm{LN}$ matrix.

Considering the whole set of $\mathrm{u}_i,\ i = 1,\dots,(\mathrm{L}-1)\mathrm{N}$ vectors we finally have

$$\mathrm{Vh} = 0. \qquad (10)$$

where $\mathrm{V} = [\mathrm{V_1}^{\mathrm{T}}, \mathrm{V_2}^{\mathrm{T}}, ..., \mathrm{V}_{(\mathrm{L}-1)\mathrm{N}}{}^{\mathrm{T}}]^{\mathrm{T}}$.

In practice we will not use the complete set of matrices $\mathrm{V}_i,\ i = 1,...,(\mathrm{L}-1)\mathrm{N}$ to define V but only a subset of it whose set of indices will be denoted by I. See (Sroubek et al., 2007) for a very interesting derivation of the above conditions and for its use in the super resolution problems see (Katsaggelos et al., 2007). See also (Gastaud et al., 2007) for the possible use of other observation models (regularization terms) for the multi-channel blur.

To use this new condition we define an additional observation model given by

$$p_2(\mathrm{g} \mid \mathrm{h},\varepsilon_{bl}) \propto \exp\left\{-\frac{1}{2}\varepsilon_{bl}\|\mathrm{Vh}\|^2\right\}, \qquad (11)$$

where ε_{bl}^{-1} is the variance of this new Gaussian observation model.

Note that

$$\|\mathrm{Vh}\|^2 = \sum_{i\in\mathrm{I}}\|\mathrm{V}_i\mathrm{h}\|^2 = \sum_{i\in\mathrm{I}}\|\sum_{j=1}^{\mathrm{L}}\mathrm{V}_i^j\mathrm{h}_j\|^2 \qquad (12)$$

In order to combine the observation model provided by Equation 5 with the observation model just described, we will use logarithmic opinion pooling techniques (Genest and Zidek, 1986) to obtain the final observation model:

$$p(\mathrm{g} \mid \mathrm{h},\mathrm{f},\beta,\varepsilon_{bl}) \propto p_1(\mathrm{g} \mid \mathrm{h},\mathrm{f},\beta)^{\lambda_1}\, p_2(\mathrm{g} \mid \mathrm{h},\varepsilon_{bl})^{\lambda_2}, \qquad (13)$$

where $\lambda_1+\lambda_2 = 1$ and $\lambda_1,\lambda_2 \geq 0$.

Note that we could have also combined both observation models using

$$\begin{aligned} p(\mathrm{g} \mid \mathrm{h},\mathrm{f},\beta,\varepsilon_{bl}) &= \lambda_1 p_1(\mathrm{g} \mid \mathrm{h},\mathrm{f},\beta) \\ &+ \lambda_2 p_2(\mathrm{g} \mid \mathrm{h},\varepsilon_{bl}) \qquad (14) \end{aligned}$$

However we will not explore this pooling of opinion technique in this paper.

In what follows we assume that each of the hyperparameters α_{im}, $\alpha_{bl,i}$, ε_{bl} and β_i are known or previously estimated and concentrate here on the estimation of the image and blur. The variational approach to be described next can incorporate the estimation of the hyperparameters but we want to concentrate here on the additional information provided by the logarithmic opinion pooling used in the observation model.

3 BAYESIAN INFERENCE

From the above definitions of the prior and observation models we have

$$\begin{aligned} -2\log \mathrm{p}(\mathrm{f},\mathrm{h},\mathrm{g}) &= \mathrm{const} \\ &+ \alpha_{im}\|\mathrm{Cf}\|^2 + \sum_{i=1}^{\mathrm{L}} \alpha_{bl,i}\|\mathrm{Ch}_i\|^2 \\ &+ \lambda_1 \sum_{i=1}^{\mathrm{L}} \beta_i \|\mathrm{g}_i - \mathrm{H}_i\mathrm{f}\|^2 \\ &+ \lambda_2\varepsilon_{bl} \sum_{i\in \mathrm{I}} \| \sum_{j=1}^{\mathrm{L}} \mathrm{V}_i^j \mathrm{h}_j\|^2 \end{aligned} \quad (15)$$

where we have removed the hyperparameters from the models because they are assumed to be known.

The Bayesian paradigm dictates that the inference on f,h should be based on the posterior distribution $\mathrm{p}(\mathrm{f},\mathrm{h},\mathrm{g})/\mathrm{p}(\mathrm{g})$. Since this posterior distribution can not be calculated in closed form we approximate it using

$$\mathrm{q}(\mathrm{f},\mathrm{h}) = \mathrm{q}(\mathrm{f})\mathrm{q}(\mathrm{h}). \quad (16)$$

The variational criterion used to find $\mathrm{q}(\mathrm{f},\mathrm{h})$ is the minimization of the Kullback-Leibler divergence, given by (Kullback and Leibler, 1951; Kullback, 1959)

$$\begin{aligned} & C_{KL}(\mathrm{q}(\mathrm{f},\mathrm{h}) \parallel \mathrm{p}(\mathrm{f},\mathrm{h}|\mathrm{g})) = \\ & \quad \int_{\mathrm{f},\mathrm{h}} \mathrm{q}(\mathrm{f},\mathrm{h}) \log\left(\frac{\mathrm{q}(\mathrm{f},\mathrm{h})}{\mathrm{p}(\mathrm{f},\mathrm{h}|\mathrm{g})}\right) d\mathrm{f}d\mathrm{h} \\ & \quad \int_{\mathrm{f},\mathrm{h}} \mathrm{q}(\mathrm{f},\mathrm{h}) \log\left(\frac{\mathrm{q}(\mathrm{f},\mathrm{h})}{\mathrm{p}(\mathrm{f},\mathrm{h},\mathrm{g})}\right) d\mathrm{f}d\mathrm{h} + \mathrm{const}, \end{aligned} \quad (17)$$

which is always non negative and equal to zero only when $\mathrm{q}(\mathrm{f},\mathrm{h}) = \mathrm{p}(\mathrm{f},\mathrm{h}|g)$.

We can then proceed to find $\mathrm{q}(\mathrm{f},\mathrm{h})$ using the following algorithm

Algorithm 1
Given $\mathrm{q}^1(\mathrm{h})$*, the initial estimate of the distribution* $\mathrm{q}(\mathrm{h})$*, for* $k = 1,2,\ldots$ *until a stopping criterion is met:*

1. *Find*

$$\mathrm{q}^k(\mathrm{f}) = \arg\min_{\mathrm{q}(\mathrm{f})} C_{KL}(\mathrm{q}(\mathrm{f})\mathrm{q}^k(\mathrm{h}) \parallel \mathrm{p}(\mathrm{f},\mathrm{h} \mid \mathrm{g})) \quad (18)$$

2. *Find*

$$\mathrm{q}^{k+1}(\mathrm{h}) = \arg\min_{\mathrm{q}(\mathrm{h})} C_{KL}(\mathrm{q}^k(\mathrm{f})\mathrm{q}(\mathrm{h}) \parallel \mathrm{p}(\mathrm{f},\mathrm{h} \mid \mathrm{g})) \quad (19)$$

The convergence of the distributions $\mathrm{q}^k(\mathrm{f})$ and $\mathrm{q}^{k+1}(\mathrm{h})$ is used as the stopping criterion of the above iterations. In order to simplify the above criterion, $\parallel \mathrm{E}[\mathrm{f}]_{\mathrm{q}^k(\mathrm{f})} - \mathrm{E}[\mathrm{f}]_{\mathrm{q}^{k-1}(\mathrm{f})} \parallel^2 / \parallel \mathrm{E}[\mathrm{f}]_{\mathrm{q}^{k-1}(\mathrm{f})} \parallel^2 < \varepsilon$, where ε is a prescribed bound, can also be used for terminating algorithm 1.

We analyze next two cases for the distributions $\mathrm{q}(\mathrm{f})$ and $\mathrm{q}(\mathrm{h})$.

3.1 Optimal random distribution for $\mathrm{q}(\mathrm{f})$ and degenerate distribution for $\mathrm{q}(\mathrm{h})$

We now proceed to explicitly calculate the distributions $\mathrm{q}^k(\mathrm{f})$ and $\mathrm{q}^{k+1}(\mathrm{h})$ in the above algorithm when we restrict the distribution of h to be of the form

$$\mathrm{q}(\mathrm{h}) = \begin{cases} 1 & \text{if } \mathrm{h} = \underline{\mathrm{h}} \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

Let us now assume that at the k-th iteration step of the above algorithm the distribution of h is degenerated on $\underline{\mathrm{h}}^k$. Then, the best estimate of the *a posteriori* conditional distribution of the real image given the observations is given by the distribution $\mathrm{q}^k(\mathrm{f})$ satisfying

$$\begin{aligned} -2\log \mathrm{q}^k(\mathrm{f}) &= \mathrm{const} + \alpha_{im} \parallel \mathrm{Cf} \parallel^2 \\ &+ \lambda_1 \sum_{i=1}^{\mathrm{L}} \beta_i \|\mathrm{g}_i - \underline{\mathrm{H}}_i^k \mathrm{f}\|^2, \end{aligned} \quad (21)$$

and thus we have that

$$\mathrm{q}^k(\mathrm{f}) = \mathcal{N}\left(\mathrm{f} \mid \mathrm{E}^k(\mathrm{f}), \mathrm{cov}^k(\mathrm{f})\right).$$

The mean of the normal distribution is the solution of

$$\frac{\partial\, 2\log \mathrm{q}^k(\mathrm{f})}{\partial \mathrm{f}} = 0,$$

while the covariance is given by

$$-\frac{\partial^2\, 2\log \mathrm{q}^k(\mathrm{f})}{\partial \mathrm{f}^2} = [\mathrm{cov}^k(\mathrm{f})]^{-1}.$$

From these two equations we obtain

$$\mathrm{E}^k(\mathrm{f}) = \left(\mathrm{M}^k(\mathrm{f})\right)^{-1} \lambda_1 \sum_i \beta_i \underline{\mathrm{H}}_i^{k\,\mathrm{T}} \mathrm{g}_i, \quad (22)$$

$$\mathrm{M}^k(\mathrm{f}) = \alpha_{im}\mathrm{C}^{\mathrm{T}}\mathrm{C} + \lambda_1 \sum_i \beta_i \underline{\mathrm{H}}_i^{k\,\mathrm{T}} \underline{\mathrm{H}}_i^k, \quad (23)$$

with

$$\mathrm{cov}^k(\mathrm{f}) \quad = \quad \left(\mathrm{M}^k(\mathrm{f})\right)^{-1}. \qquad (24)$$

Once $\mathrm{q}^k(\mathrm{f})$ has been calculated $\underline{\mathrm{h}}^{k+1}$ satisfies

$$\lambda_1\beta_j\mathrm{E}^k(\mathrm{F})^{\mathrm{T}}\mathrm{g}_j = \lambda_2\varepsilon_{bl}\sum_{i\in \mathrm{I}}(\mathrm{V}_i^j)^{\mathrm{T}}(\sum_{l=1}^{\mathrm{L}}\mathrm{V}_i^l\underline{\mathrm{h}}_l^{k+1})$$
$$+[\alpha_{bl,j}\mathrm{C}^{\mathrm{T}}\mathrm{C}+\lambda_1\beta_j\mathrm{E}^k(\mathrm{F})^{\mathrm{T}}\mathrm{E}^k(\mathrm{F})+$$
$$\lambda_1\beta_j\mathrm{N}\mathrm{cov}^k(\mathrm{f})]\underline{\mathrm{h}}_j^{k+1}, \qquad j=1,\ldots,\mathrm{L} \quad (25)$$

Rewriting Equation 25 in a more compact form we obtain

$$\underline{\mathrm{h}}^{k+1} =$$
$$\left[\mathrm{K}(\alpha_{bl},\mathrm{C}^{\mathrm{T}}\mathrm{C})+\right.$$
$$\lambda_1\mathrm{K}\left(\beta,\mathrm{E}^k(\mathrm{F})^{\mathrm{T}}\mathrm{E}^k(\mathrm{F})+\mathrm{N}\mathrm{cov}^k(\mathrm{f})\right)+$$
$$\left.\lambda_2\varepsilon_{bl}\mathcal{V}_{\mathrm{I}}^{\mathrm{T}}\mathcal{V}_{\mathrm{I}}\right]^{-1}\times\begin{pmatrix}\lambda_1\beta_1\mathrm{E}^k(\mathrm{F})^{\mathrm{T}}\mathrm{g}_1\\ \lambda_1\beta_2\mathrm{E}^k(\mathrm{F})^{\mathrm{T}}\mathrm{g}_2\\ \vdots\\ \lambda_1\beta_{\mathrm{L}}\mathrm{E}^k(\mathrm{F})^{\mathrm{T}}\mathrm{g}_{\mathrm{L}}\end{pmatrix}.(26)$$

Note that $\mathcal{V}_{\mathrm{I}}$ is defined by $\mathcal{V}_{\mathrm{I}}=[\mathrm{V}_{i_1}{}^{\mathrm{T}},\mathrm{V}_{i_2}{}^{\mathrm{T}},\ldots,\mathrm{V}_{i_{|\mathrm{I}|}}{}^{\mathrm{T}}]^{\mathrm{T}}$, where $|\mathrm{I}|$ denotes cardinality of set I. Matrix $\mathrm{K}(\mathrm{x},\mathrm{Y})$ is defined with the help of the Kronecker product operator $\otimes$ as $\mathrm{K}(\mathrm{x},\mathrm{Y}) = \mathrm{Diag}(\mathrm{x})\otimes\mathrm{Y}$. Here, matrix $\mathrm{Diag}(\mathrm{x})$ represents a diagonal matrix with its main diagonal elements in the same order as the elements of the vector x.

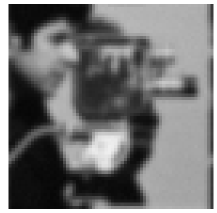
3.2 Optimal degenerate distributions for $\mathrm{q}(\mathrm{f})$ and $\mathrm{q}(\mathrm{h})$

In order to obtain the best degenerate distributions for the image and blur we simply have to use $\mathrm{cov}^k(\mathrm{f}) = 0$ in Equation 25 and use $\underline{\mathrm{f}}^k = \mathrm{E}^k(\mathrm{f})$ where the expected value $\mathrm{E}^k(\mathrm{f})$ has been defined in Equation 22.

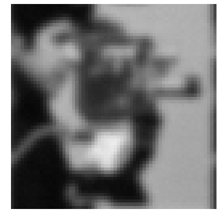
4 EXPERIMENTAL RESULTS

In this section experimental results with the proposed BMCR algorithms are shown. We will examine the performance of our proposed algorithms using two sets of four distorted observations of the original scene. Each set of observed images was obtained by blurring the original scene with Gaussian blurs with variances 1,2,3,4 and adding Gaussian noise to each channel so that their Blurred Signal to Noise Ratio (BSNR) was equal to 40dB and 20dB. Observations for the 40dB BSNR case are shown in Figure 1.

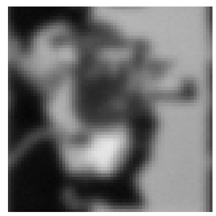
In order to compare different restorations we have used Improved Signal to Noise Ratio (ISNR) as our comparison metric. ISNR is defined as $10\log_{10}\left(\|\mathrm{g}_i-\mathrm{f}\|^2/\|\mathrm{f}^k-\mathrm{f}\|^2\right)$.



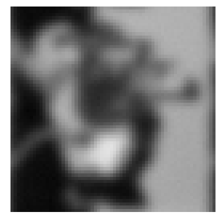
(a) Channel 1.



(b) Channel 2.



(c) Channel 3.



(d) Channel 4.

Figure 1: Multi-channel observations (BSNR=40dB).

In order to obtain an upper bound for our blind multi-channel restoration results, we performed non blind multi-channel restoration. This restoration results from setting $\alpha_{bl,i} = \lambda_2 = 0$ in Equation 15. The non blind multi-channel based restoration for the BSNR=40dB observation set is shown in Figure 2 and the corresponding ISNR values in dB are shown in Table 1.

Figure 2: Non blind multi-channel restoration (BSNR = 40dB).

Table 1: Non blind restoration ISNR values in dB.

Channel No.	BSNR = 40dB	BSNR = 20dB
1	9.27	5.08
2	9.17	5.02
3	9.35	5.11
4	9.55	5.32

In order to better understand and to quantify the information provided by each prior and observation model we normalized the parameters so that $p_1+p_2+p_3+p_4=1$, where $p_1=\alpha_{im}$, $p_2=\lambda_2\varepsilon_{bl}$, $p_3=\sum_i \alpha_{bl,i}$ and $p_4=\sum_i \lambda_1\beta_i$.

We proceeded by setting p_2 equal to zero and by adjusting p_1 and p_3 to maximize the ISNR of the restoration. Once p_1 and p_3 were determined, the values of p_2 and p_4 were varied to determine the significance of the observation models used in the restoration process.

Tables 2 and 3 show that the ISNR of the restoration obtained by combining the noise and subspace observation models is greater than the one obtained by using only the noise observation model. For the BSNR=40dB set, the improvement on average is 0.66dB and 1.17dB depending on whether we use non degenerate or degenerate distributions to approximate the image posterior distribution. This table also shows that the approximation of the global posterior distribution by a combination of degenerate distribution for the blur and non degenerate distribution for the image outperforms the model where both are degenerate.

It is important to understand what is the value that multiple observations of the same scene bring to the blind restoration problem. In order to answer this question we performed blind single channel restoration on the least blurred channel. Tables 4 and 5 show the corresponding blind single-channel restoration results. It can be observed that our blind multi-channel based restoration with optimal random distribution q(f) outperforms blind-single channel restoration as well. However, this is not the case for the blind multi-channel restoration based on the degenerate random distribution q(f).

It can also be observed that our best blind-multi channel based restoration for the BSNR=40dB set is approximately 2.4dB below its upper bound from Table 1. The blind multi-channel based restoration with optimal random distribution q(f) for the BSNR=40dB observation set is shown in Figure 3.

Figure 3: Blind multi-channel restoration (BSNR = 40dB) with optimal random distribution q(f).

Table 2: Blind multi channel restoration for the BSNR=40dB with optimal random distribution q(f).

(p_1,p_2,p_3,p_4)	ISNR [dB]
(1e-4,0,0.12,0.8799)	Ch. 1: 6.22 Ch. 2: 6.12 Ch. 3: 6.30 Ch. 4: 6.51
(1e-4,0.5,0.12,0.3799)	Ch. 1: 6.88 Ch. 2: 6.78 Ch. 3: 6.96 Ch. 4: 7.16

Table 3: Blind multi channel restoration for the BSNR=40dB with degenerate random distribution q(f).

(p_1,p_2,p_3,p_4)	ISNR [dB]
(1e-4,0,0.12,0.8799)	Ch. 1: 3.18 Ch. 2: 3.08 Ch. 3: 3.26 Ch. 4: 3.46
(1e-4,0.1,0.12,0.7799)	Ch. 1: 4.35 Ch. 2: 4.25 Ch. 3: 4.43 Ch. 4: 4.63

Table 4: Blind single channel restoration for the BSNR=40dB with optimal random distribution q(f).

(p_1,p_2,p_3,p_4)	ISNR [dB]
(1e-4,0,0.12,0.8799)	Ch. 1: 5.98

Table 5: Blind single channel restoration for the BSNR=40dB with degenerate random distribution q(f).

(p_1,p_2,p_3,p_4)	ISNR [dB]
(1e-4,0,0.12,0.8799)	Ch. 1: 4.51

5 CONCLUSIONS

In this paper we examined the use of the logarithmic opinion pooling to statistically combine two observation models that are regularly used in the multichannel image restoration algorithms. In order to provide an estimate of the posterior distributions of the real underlying image and the unknown blurs variational techniques were used. Variational approximations lead to two different iterative blind multichannel based restoration algorithms. Both of these algorithms are incorporating some prior assumptions (e.g. SAR models on unknown image and blurs) about the unknowns into the restoration process. Additionally, both algorithms are incorporating two observations models into the restoration process, which allows us to further constraint unknown blurs in particular. As can be seen from the experimental section both multi-channel based restoration algorithms performed better when logarithmic opinion pooling technique was used to statistically combine observation models into the restoration process.

ACKNOWLEDGEMENTS

This work has been supported by the Comisión Nacional de Ciencia y Tecnología under contract TIC2007-65533.

REFERENCES

Bishop, T., Babacan, D., Amizic, B., Katsaggelos, A. K., Chan, T., and Molina, R. (2007). Blind image deconvolution: problem formulation and existing approaches. In Campisi, P. and Egiazarian, K., editors, *Blind image deconvolution: Theory and Applications*, chapter 1, pages 1–42. CRC.

Galatsanos, N. P., Mesarovic, V. Z., Molina, R., Katsaggelos, A. K., and Mateos, J. (2002). Hyperparameter estimation in image restoration problems with partially-known blurs. *Optical Eng.*, 41(8):1845–1854.

Gastaud, M., Ladjal, S., and Matre, H. (2007). Blind filter identification and image superresolution using subspace methods. In *European Signal Processing Conference*. Poznan (Poland).

Genest, C. and Zidek, J. V. (1986). Combining probability distributions: A critique and an annotated bibliography. *Statistical Science*, 1:114148.

Katsaggelos, A., Molina, R., and Mateos, J. (2007). *Super resolution of images and video*. Synthesis Lectures on Image, Video, and Multimedia Processing. Morgan and Claypool.

Kullback, S. (1959). *Information Theory and Statistics*. New York, Dover Publications.

Kullback, S. and Leibler, R. A. (1951). On information and sufficiency. *Annals of Mathematical Statistics*, 22:79–86.

Mateos, J., Katsaggelos, A., and Molina, R. (2000). A Bayesian approach to estimate and transmit regularization parameters for reducing blocking artifacts. 9(7):1200–1215.

Molina, R., Katsaggelos, A. K., and Mateos, J. (1999). Bayesian and regularization methods for hyperparameter estimation in image restoration. 8(2):231–246.

Molina, R., Mateos, J., and Katsaggelos, A. (2006). Blind deconvolution using a variational approach to parameter, image, and blur estimation. *IEEE Trans. on Image Processing*, 15(12):3715–3727.

Ripley, B. D. (1981). *Spatial Statistics*, pages 88–90. John Wiley.

Sroubek, F., Cristóbal, G., and Flusser, J. (2007). A unified approach to superresolution and multichannel blind deconvolution. *IEEE Transactions on Image Processing*, 9:2322–2332.