Super resolution and pansharpening of Multispectral Images

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Outline

I. Super resolution in Remote Sensing
II. Super resolution methods in Remote Sensing
   I. Method of Akgun et al.
   II. Method of Price
   III. Method of Eismann et al.
III. A new SR method in Remote Sensing
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I Super resolution in Remote Sensing

With an ideal sensor we would have high resolution multispectral images.

Unfortunately due to spectral and spatial decimation we have:

Spectral decimator

Spatial decimator
Observed high resolution panchromatic image (x)

A band (y^b) of the observed low resolution multispectral image (Y^b) we want to estimate

High resolution multispectral image (y) we want to estimate

NOTATION

Upper case: low resolution
Y(i) = (Y^1(i), ..., Y^K(i))^T

Lower case: high resolution
y(j) = (y^1(j), ..., y^L(j))^T
II Super resolution methods in Remote Sensing

II.I Method of Akgun et al.

$f^\lambda(x)$ denotes a high resolution image at $\lambda$ wavelength.

$f^\lambda(x)$ can be decomposed as

$$f^\lambda(x) = \sum_{j=1}^{P} b_j(\lambda) f_j(x)$$

- **Known quantities**
- **High resolution images to be estimated**
Comments on the coefficients $b_j(\lambda)$ and the decomposition:

With an example, If $P=2$ and

$$\lambda \in [0, \lambda_{\text{max}}]$$

We could use

$$b_1(\lambda) = \begin{cases} 1 & \text{if } \lambda < \lambda_{\text{max}} / 2 \\ 0 & \text{elsewhere} \end{cases} \quad b_2(\lambda) = \begin{cases} 1 & \text{if } \lambda \geq \lambda_{\text{max}} / 2 \\ 0 & \text{elsewhere} \end{cases}$$

and we would have to estimate $f_1(x)$ and $f_2(x)$

We could also use PCA
What observations do we have?

Remember

\[ f^\lambda (x) = \sum_{j=1}^{P} b_j (\lambda) f_j (x) \]

First, each \( f^\lambda \) is convolved with a convolution filter \( H \) producing

**Spatial filtering**

\[ f^{b,\lambda} = Hf^\lambda = \sum_{j=1}^{P} b_j (\lambda) Hf_j \]

Then, \( f^{b,\lambda} \) is weighted (integrated) on \( \lambda \) obtaining

**Spectral filtering**

\[
\begin{align*}
  f^{w,b} &= \sum_{\lambda} \eta(\lambda) f^{b,\lambda} = \sum_{\lambda} \eta(\lambda) Hf^\lambda = \sum_{\lambda} \eta(\lambda) \left( \sum_{j=1}^{P} b_j (\lambda) Hf_j \right) \\
  &= \sum_{j=1}^{P} w_j Hf_j
\end{align*}
\]
Finally, $f^{w,b}$ is decimated to produce the observed image

$$g = D\left(\sum_{\lambda} \eta(\lambda) f^{b,\lambda}\right) = D\left(\sum_{\lambda} \eta(\lambda) Hf^{\lambda}\right)$$

$$= D\left(\sum_{\lambda} \eta(\lambda) \left(\sum_{j=1}^{P} b_j(\lambda) Hf_j\right)\right)$$

$$= D\left(\sum_{j=1}^{P} w_j Hf_j\right) = \sum_{j=1}^{P} w_j DHf_j$$

Usually we do not have just one low resolution image but a hypercube of low resolution observations and the above equation can be written for each band $g_i$ of the hypercube. We have
Gaussian independent noise is usually included in the model so we have

\[ g = \left( \begin{array}{c} g_1 \\ \vdots \\ g_Q \end{array} \right) \]

where for each image \( i \) in the hypercube we have

\[ g_i = \sum_{j=1}^{P} w_{i,j} \cdot DHf_j + \epsilon_i \]

\( i = 1, \ldots, Q \)

POCS with additional constraints is used to estimate \( f_j \) \( j=1,\ldots,P \)
Instead of having just one hypercube

We may have several hypercubes

$$g = \begin{pmatrix} g_1 \\ \vdots \\ g_Q \end{pmatrix}$$

$$g^l, \quad l = 1, \ldots, R$$

We could then register all hypercubes with respect to one, for instance $g^1$, and we would obtain

$$g^l = \sum_{j=1}^{P} w_{i,j} DHR_{(l,1)} f_j + \varepsilon_i^l \quad i = 1, \ldots, Q$$

- The weights could be hypercube dependent
- The blur could be hypercube dependent
- Band in hypercube $l$
- Registration between hypercubes $l$ and 1
- Hypercube $l$
II.II Method of Price

It is assumed that a high resolution panchromatic image \( x \) and a low resolution multispectral image \( Y \) are available.

We will assume, for simplicity a 2x2 magnifying factor.

\((i,j)\) denotes low resolution pixel. This low resolution pixel consists of four high resolution pixels \((u,v)\) with
\[(u,v) \in H_{ij} = \{(2i,2j), (2i+1,2j), (2i,2j+1), (2i+1,2j+1)\}.

\[
X(i, j) = \frac{1}{4} \sum_{(u,v) \in H_{ij}} x(u, v)
\]

Panchromatic low resolution image
**Price’s model**


For each band $b$ and for $(u,v) \in H_{ij}$ the following assumption is used

$$y^b(u,v) - Y^b(i,j) = a^b_{i,j} (x(u,v) - X(i,j))$$

Both values have to be estimated

First, we estimate $a^b_{i,j}$ and then we calculate $y^b(u,v)$ using the above equation.
Estimating $a_{i,j}^b$ in

$$y^b(u,v) - Y^b(i,j) = a_{i,j}^b (x(u,v) - X(i,j))$$

1. $y^b(u,v)$ and $x(u,v)$ are replaced by $Y^b(p,q)$ and $X(p,q)$, respectively, where $(p,q) \in L_{i,j}$ (a set of low resolution neighboring pixels of pixel $(i,j)$, usually 3x3)

2. and then

$$a_{i,j}^b = \arg\min_a \sum_{(p,q) \in L_{i,j}} (Y^b(p,q) - Y^b(i,j) - a(X(p,q) - X(i,j)))^2$$

which produces

$$a_{i,j}^b = \frac{\sum_{(u,v) \in L_{i,j}} [Y^b(u,v) - Y^b(i,j)][X(u,v) - X(i,j)]}{\sum_{(u,v) \in L_{i,j}} [X(u,v) - X(i,j)]^2}$$

We now estimate $y^b(u,v)$ using

$$y^b(u,v) - Y^b(i,j) = a_{i,j}^b (x(u,v) - X(i,j))$$
Extension:
J.H. Park and M.G. Kang, “Spatially adaptive multi-resolution multispectral image fusion”,

Let us consider again Price’s equation

\[ y^b(u,v) - Y^b(i,j) = a^b_{i,j} (x(u,v) - X(i,j)) \]

Then, Park and Kang consider for each high resolution pixel 
(p,q)

\[ y^b(p,q) - \bar{y}^b(p,q) = a^b_{u,v} (x(p,q) - \bar{x}(p,q)) \]

where

\[ \bar{x}(p,q) = \frac{1}{4} \sum_{(u,v) \text{ neighbors of } (p,q)} x(u,v) \]
\[ \bar{y}^b(p,q) = \frac{1}{4} \sum_{(u,v) \text{ neighbors of } (p,q)} y^b(u,v) \]
In order to estimate $a_{u,v}^b$ in

$$y^b(p,q) - \bar{y}^b(p,q) = a_{u,v}^b (x(p,q) - \bar{x}(p,q))$$

The following procedure is proposed:

- $x(p,q)$ is replaced by its downsampled (to the size of the low resolution images) and then upsampled version to its original size. The new value is denoted by $x'(u,v)$. Then $\bar{x}'(u,v)$ is calculated.
- $y^b(u,v)$ is estimated as an upsampled version of $Y^b$. The new value is denoted $y'^b$ and $\bar{y}'^b$ is now calculated.

and $a_{u,v}^b$ is estimated from

$$y'^b(p,q) - \bar{y}'^b(p,q) = a_{u,v}^b (x'(p,q) - \bar{x}'(p,q))$$
as

\[ a^{b}_{p,q} = \]

\[
\arg\min_a \sum_{(u,v) \text{ neighbors of } (p,q)} w_{u,v} \left( y^{ib}(u,v) - \bar{y}^{ib}(u,v) - a(x'(u,v) - \bar{x}'(u,v)) \right)^2
\]

where \( w_{u,v} \) is a similarity measure between pixels \((u,v)\) and \((p,q)\).

Finally, \( y^b(p,q) \) is calculated using

\[
y^b(p,q) = a^{b}_{p,q} (x(p,q) - \bar{x}(p,q)) + \bar{y}^b(p,q)
\]
II.III Method of Eismann et al

The panchromatic high resolution image $x$ can be written as

$$x = S^t y + \eta$$

where $y$ is the high resolution multispectral image we want to estimate, $S$ is a sparse matrix whose rows are the spectral response functions for the panchromatic pixel locations and $\eta$ is the noise. The above equation produces $P(x|y)$.

The low resolution observations $Y$ can be expressed as

$$Y = Hy + \epsilon$$

where $\epsilon$ is the noise and $H$ is a sparse matrix whose rows are the spatial response functions for the low resolution hyperspectral pixels. The above equation produces $P(Y|y)$. 
Using the Bayesian paradigm, our goal becomes finding the Maximum a Posteriori (MAP), that is

\[
\hat{y} = \arg \max_y P(y | x, Y)
\]

where we have

\[
P(y | x, Y) \propto P(y) P(x, Y | y)
\]

assuming independence between \(x\) and \(Y\) given \(y\) we write

\[
P(y | x, Y) \propto P(y) P(x | y) P(Y | y)
\]

or

\[
P(y | x, Y) \propto P(y | x) P(Y | y)
\]

The only remaining task is the definition of \(P(y)\) or the conditional distribution \(P(y | x)\) depending on the model we want to use.
Using the model

\[ P(y \mid x, Y) \propto P(y \mid x)P(Y \mid y) \]


The authors propose to estimate \( P(y\mid x) \) using a joint Gaussian distribution for \((y,x)\) and then calculate the conditional.

Mean and Covariance matrices are obtained from the panchromatic and low resolution images. Covariance matrices are improved by the use of clustering techniques.
Using the model

\[ P(y \mid x, Y) \propto P(y)P(x \mid y)P(Y \mid y) \]


\( P(y) \) is estimated for each pixel as a mixture of Gaussian distributions and the mean and covariance of each member of the mixture is estimated using the Stochastic Mixing Model (SMM), see paper for details. The element of the mixture with the highest probability defines then the prior model.

Note that we can also use the SMM when estimating \( P(y \mid x) \).

III. A new SR model in Remote Sensing

We assume that a high resolution panchromatic image $x$ and a low resolution multispectral image $Y$ are available. We want to obtain a high resolution hypercube $y$.

$(i, j)$ denotes low resolution pixel. This low resolution pixel consists of four high resolution pixels $(u,v)$ with $(u,v) \in H_{ij} = \{(2i,2j), (2i+1,2j), (2i,2j+1), (2i+1,2j+1)\}$.

\[ Y^b(i, j) = \frac{1}{4} \sum_{(u,v) \in H_{ij}} y^b(u, v) = (Hy^b)(i, j) \]

Low resolution band from its corresponding high resolution band
Let us assume that we have B observed low resolution images \( Y^1, \ldots, Y^B \) and a high resolution panchromatic image \( x \).

We want to estimate the corresponding B high resolution images \( y^1, \ldots, y^B \) with the use of the information provided by the low resolution observations and the panchromatic image.

Let us denote by \( y \) the whole set of high resolution images \( y^1, \ldots, y^B \) we want to estimate.

The process to obtain the low resolution observations for the high resolution images we want to estimate is modeled by

\[
P(Y^b \mid y) = P(Y^b \mid y^b) \propto \exp\left[ -\frac{\alpha_b}{2} \| Y^b - DHy^b \|^2 \right]
\]

where \( D \) models the downsampling operation
The panchromatic image is formed as a linear combination of the high resolution hypercube bands plus additive noise:

\[ x(u, v) = \sum_b \lambda^b y^b(u, v) + \varepsilon(u, v) \]

\( \lambda^b \geq 0 \) are known quantities weighting the contribution of each high resolution band we want to estimate to the high resolution panchromatic image.

There is work to be done on the estimation of these weights. Blind deconvolution techniques?
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**LandSat ETM+ Spectral Response**

<table>
<thead>
<tr>
<th>Color</th>
<th><strong>LANDSAT ETM+ band</strong></th>
<th>Color</th>
<th><strong>LANDSAT ETM+ band</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>1 (0.45 µm to 0.515 µm)</td>
<td>Yellow</td>
<td>5 (1.55 µm to 1.75 µm)</td>
</tr>
<tr>
<td>Red</td>
<td>2 (0.525 µm to 0.605 µm)</td>
<td>Not shown</td>
<td>6 (10.4 µm to 12.5 µm)</td>
</tr>
<tr>
<td>Green</td>
<td>3 (0.63 µm to 0.69 µm)</td>
<td>Cyan</td>
<td>7 (2.08 µm to 2.35 µm)</td>
</tr>
<tr>
<td>Blue</td>
<td>4 (0.75 µm to 0.9 µm)</td>
<td>Magenta</td>
<td>Pan (0.51 µm to 0.9 µm)</td>
</tr>
</tbody>
</table>
The panchromatic image provides no information on these two bands

We intend to reconstruct all the $B$ bands $y^b$, $b=1,...,B$ simultaneously. For Lansat ETM+ images we have three bands to be reconstructed. So, in this case $B=4$.

We assume

$$P(x | y^1, ..., y^B) \propto \exp \left[ -\frac{\alpha}{2} \| x - \sum_{j=1}^{B} \lambda^j y^j \|_2^2 \right]$$
A priori we assume that all high resolution images are smooth and no correlation between them exists (this needs more work), so we write

$$P(y) = \prod_{b=1}^{B} P(y^b) \propto \prod_{b=1}^{B} \exp\left[-\frac{\beta_b}{2} \|Cy^b\|^2\right]$$

where $C$ denotes the Laplacian operator.

We now use the Bayesian paradigm and write
\[ P(y | x, Y) \propto P(y)P(x, Y | y) \]

\[ = \left( \prod_{b=1}^{B} P(y^b) \right)P(x, Y | y) \]

\[ = \left( \prod_{b=1}^{B} P(y^b) \right)P(x | y)P(Y | y) \]

\[ = \left( \prod_{b=1}^{B} P(y^b) \right)P(x | y)\left( \prod_{b=1}^{B} P(Y^b | y^b) \right) \]

Our goal then becomes finding

\[ \hat{y} = \arg \max_y \left( \prod_{b=1}^{B} P(y^b) \right)P(x | y)\left( \prod_{b=1}^{B} P(Y^b | y^b) \right) \]

Not very realistic
\[
\hat{y} = \arg\min_y \left\{ \sum_b \alpha_b \| Y^b - Dh^j \|^2 + \alpha \left\| x - \sum_j \lambda^j y^j \right\|^2 + \sum_b \beta_b \| C^b \|^2 \right\}
\]

Fidelity to low resolution observations
Fidelity to the panchromatic image
Smoothness constraints

Because of the form of the function to be optimized (of the involved matrices), its solution can be found using non-iterative techniques.

Note also that the unknown parameters can be estimated using the E-M algorithm (work in progress).
IV. Examples

panchromatic

Low resolution bands 1 to 4
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Low resolution bilinearly interpolated

Proposed method

Price’s method

band 1
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Low resolution bilinearly interpolated

Proposed method

Price’s method

band 2
Price's method

Low resolution bilinearly interpolated

Proposed method
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Low resolution bilinearly interpolated

Proposed method

Price’s method

band 4
R = band 3
G = band 2
B = band 1

Price´s method

Proposed method

Low resolution bilinearly interpolated
R = band 3
G = band 4
B = band 2

Low resolution bilinearly interpolated

Price's method

Proposed method
Low resolution band 1

Reconstructed band 1
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--- Low resolution band 4

--- Reconstructed band 4
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\[ \sum_{b} \lambda^{b} \hat{y}^{b}(u, v) \]
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panchromatic

Low resolution bands 1 to 4
R = band 3
G = band 2
B = band 1

Low resolution bilinearly interpolated

Price´s method

Proposed method
R = band 4
G = band 3
B = band 2

Price’s method

Proposed method

Low resolution bilinearly interpolated

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V. Conclusions

Super resolution methods in Remote Sensing have been described.

A new super resolution method in Remote Sensing has been proposed.

Some preliminary examples have been shown.