On Automatic Selection of Fuzzy Homogeneity Measures for Path-based Image Segmentation

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Abstract

This paper is the continuation of a previous work were we proposed the use of t-norms to measure path homogeneity in path-based fuzzy image segmentation. Here, we introduce an approach to automatically select the homogeneity function, taking into account the characteristics of the region to be segmented and its contour. To this purpose we firstly approximate a characteristic of the region surrounding the seed, on the basis of the study of a set of fixed paths. Secondly, we study the relationship between this characteristic and the parameter that controls the behavior of homogeneity functions based on Weber's t-norm. Finally, we obtain a function that from the value of the computed slope, gives the value of the parameter whose associated homogeneity function is suitable to model the region.

Keywords: Image segmentation, fuzzy segmentation, path-based segmentation, fuzzy connectivity, fuzzy color homogeneity.

1 Introduction

Image segmentation is the process of partitioning the image into connected subsets of pixels, called regions, on the basis of some homogeneity criteria. It is used in many image analysis techniques and applications, like image database retrieval or robot vision [2], since segmented region are their starting point.

There are several segmentation techniques that provide a crisp segmentation of images, where each pixel belongs to an unique region, what is not often suitable to model imprecise regions, usually found in natural images, as occurs in shadows, brights and color gradients. This is why fuzzy segmentation techniques arise, defining a *region* as a fuzzy subset of pixels, such that every pixel in the image has a membership degree to that region [1, 3]. However, not all these techniques take into account that a region must be topologically connected.

To face up this problem, path-based techniques arise, incorporating spatial information related to adjacency between pixels. Based on the idea of fuzzy topology, introduced by Rosenfeld [8], and the use of fuzzy connectivity to measure the relationship between any pair of pixels, these techniques compute this connectivity regarding the seed of a region, and obtain the fuzzy region [7, 1, 9, 6, 5]. This fuzzy connectivity and so, the membership degree, is based on the an homogeneity measure of a "path" connecting two pixels.

Most of the proposed homogeneity functions are based on the aggregation of the distances between consecutive pixels [7, 1, 4], and obtaining homogeneity as a kind of inverse of heterogeneity. In this context, an open problem is, given the seed of a region, how to select an homogeneity function suitable to model the fuzzy region. In real images there is a great variety of regions, with different homogeneity and contour characteristics, and for each kind of region a different homogeneity function should be used: the fuzzier the region and its contour is, the softer the homogeneity function should be.

As we saw in [4], t-norms may be used as homogeneity functions, offering a great variety of behaviors, specially parametric t-norms. It allows us to model any kind of fuzzy region. Hence, the next problem posed in that paper was to establish a relationship between homogeneity functions and region characteristics, that allowed us to automatically determine which homogeneity function is suitable to model a given region.

In this paper, after summarizing a methodology for path-based image segmentation in section 2, we approximate a feature of the region surrounding a seed, in section 3.1. This value is used in the selection of the homogeneity function. To this purpose, in section 3.2, we study the relationship between this slope and the parameter of a t-norm offering a wide range of homogeneity functions. In this way a function to get the parameter value from approached seed surrounding region's slope, is obtained. With it, the process to select the homogeneity function suitable to model the fuzzy region is automated. In sections 4 and 5 we show the results obtained applying this automatization and its conclusions.

2 Path Based Segmentation

In this section we summarize the path-based approach presented in [4]that incorporates spatial information about pixel adjacency by measuring the connectivity between any pair of pixels, as the homogeneity of the most homogeneous path joining them. Based on it, fuzzy image segmentation is performed computing the connectivity between each seed point and all the pixels in the image.

2.1 Pixel Characterization

Firstly, to characterize a pixel, p, in the image, we use a feature vector, $\overrightarrow{f_p}$, where each feature, f_p^i, is a numerical measure of any relevant characteristic obtained for p. In our case, we are interested on obtaining homogeneously-colored regions, so we have chosen the human perception based color space HSI (hue, saturation or purity and intensity or lightness) [4]. Hence, the feature vector we use to characterize each pixel p in the image, contains its three band color representation in the HSI color space, as in equation 1:

$$\overrightarrow{f_p} = [H_p, S_p, I_p] \tag{1}$$

2.2 Fuzzy Resemblance Between Pixels

Once we have characterized each pixel, we use in [4] a resemblance relation, $\mathcal{FR}$, defined on above mentioned feature vectors, as a measure of how resemblant two pixels are. Calculation of resemblance measure between features vectors depends on the concrete features employed. In our case, feature vectors resemblance is based on color resemblance that is, as well, based on the distance in HSI color space, $\triangle C(c_1, c_2)$. This distance is defined in [4] on the basis of the differences in [0,1] between color components, $\triangle_H$, $\triangle_S$ and $\triangle_I$ for hue, saturation and intensity values, respectively.

On the basis of this distance between colors, we define the resemblance between the feature vectors f_p and f_q corresponding to pixels p and q in the image IM as in equation 2

$$\mathcal{FR}(\overrightarrow{f_p}, \overrightarrow{f_q}) = 1 - \triangle C(\overrightarrow{f_p}, \overrightarrow{f_q}) \tag{2}$$

Finally, we define a resemblance relation $\mathcal{PR}$ between neighbor pixels, as the relation $\mathcal{FR}$ between their corresponding feature vectors, in this way:

$$\mathcal{PR}(p, q) = \mathcal{FR}(\overrightarrow{f_p}, \overrightarrow{f_q}) \tag{3}$$

2.3 Fuzzy Connectivity Between Pixels

So far, we can know the resemblance between two adjacent pixels. To measure resemblance between any two pixels, we use the fuzzy connectivity measure. Fuzzy connectivity of two pixels indicates, in fuzzy path-based image segmentation, the degree to which those pixels belong to a group of topologically connected pixels with resemblant features. Therefore to measure the fuzzy connectivity between two pixels we use information about the homogeneity of the paths joining them.

Given two pixels, p and q, we defined in [4] the path joining them, π_{pq}, as the sequence:

$$\pi_{pq} = (r_1, r_2, \ldots, r_k) \tag{4}$$

where $k \geq 1$, such that $r_1 = p$ and $r_k = q$ and r_i is connected to r_{i+1} $\forall i \in \{1, \ldots, k-1\}$. With Π_{pq} we note the set of all the possible paths between p and q.

In addition, we defined the homogeneity of each path joining p and q as a function, $homo : \Pi_{pq} \rightarrow [0, 1]$, calculated on the basis of resemblance between consecutive points in the path.

In [4] we studied the characteristics that an homogeneity function should verify, and we concluded that is was a natural way to define them as an aggregation of resemblances between pairs of consecutive points in the path. Since t-norms are a natural way to aggregate values, behavior of bounded difference, algebraic product and also t-norms of Frank, Dubois and Prade and Weber was studied and compared. In the last three cases we have parametric functions, that show different behaviors depending on the value of their parameter. These different behaviors corresponds to different homogeneity functions that make t-norms able to adapt to different situations and types os regions.

Taking the *homo* function into account, we define the optimum path between p and q, $\hat{\pi}_{pq}$, as the path linking them with maximum homogeneity. Based on it, we get the measure of the connectivity between two pixels as the homogeneity value of the optimum path joining them:

$$conn(p, q) = homo(\hat{\pi}_{pq}) \qquad (5)$$

2.4 Membership Functions for Fuzzy Regions

The aforementioned fuzzy connectivity between two pixels lets us define, in equation 6, the membership degree of a pixel p, to a region $\widetilde{R_s}$, as the connectivity between the pixel and the seed point, r_s, of the region.

$$\mu_{\widetilde{R_s}}(p) = conn(p, r_s) \qquad (6)$$

Computing the membership degree of each point p in the image to each region $\widetilde{R_s}$, we obtain the set of fuzzy regions resulting of the fuzzy segmentation process, with a computational cost of $O(mn)$, as detailed in [2]. In these segmentation process and open problem is the selection of the homogeneity function for each region, depending on its homogeneity characteristics.

3 Automatic Selection of Homogeneity Functions

In the previous section we summarized a methodology for fuzzy path-based image segmentation, pointing automatic homogeneity function selection as unsolved topic. In this section we propose an approach to, firstly, in section 3.1, identify characteristics of region surrounding the seed, based of its first border's slope and, secondly, in section 3.2ind the most suitable homogeneity function to model that region.

3.1 Seed's Surroundings Characterization

In real images we can find several types of regions according to their homogeneity and contour characteristics. Since we are interested on homogeneously colored regions, we assume that regions we work on, are homogeneous regarding their color. Then, differences between different types of regions are in their borders. Having it, regions vary between two extreme situations: on one hand, we have homogeneous regions with well defined, and even, crisp contours; on the other hand, there are regions with very soft changes from the color inside the region to the one outside, being the most representative case a soft tone down region. Between these extreme situations, there are a great variety of regions, where crisp contours become wider and softer.

Therefore, to find a suitable homogeneity function to model the region, we must study region's contours. We need to obtain a value representative of border's variation, telling wether it is thin and with great variation in pixel's features, or wide with small pixel's features changes. The value that gives this information is the slope of the border, so here we try to find this border and approximate the value of its slope.

Given a seed point r_s (blue cross in 1-A) for the fuzzy region $\widetilde{R_s}$, we study a set of paths starting at the seed, and with different directions around it, to determine the characteristics of the area surrounding the seed. Then, we study each path and aggregate, in the approached region's slope, the information obtained.

Depending on the set of paths used, the infor-

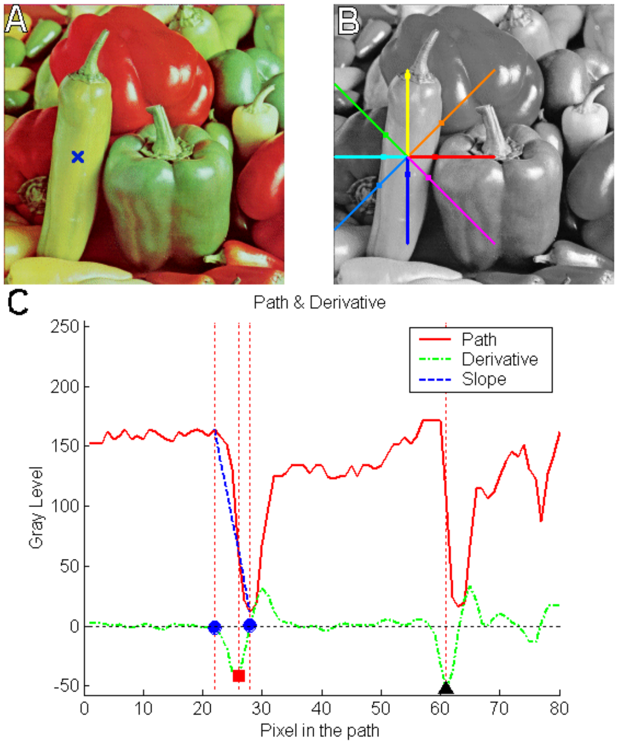

Figure 1: Process to approximate region's slope. A: the original image with the seed. B: Set of studied paths. C: Profile, derivative, gradient, points and rect used to approach region's slope.

mation we work with will be more or less representative of region's borders. Intuitively, these paths should be straight lines homogeneously distributed through different directions around the seed, and have a length according to region's size. An example of these paths is shown in 1-B.

For each path, π_{sq}, we obtain its profile, defined as the gray levels (intensity component in HSI) of each pixel along the path. This profile is a function in equation 7:

$$Profile_{\pi_{sq}}(r_k) = I(r_k) \tag{7}$$

with $r_k \in \pi_{sq}$, and I is the intensity value for each pixel in the image. Intuitively, this function tells how the region is in the direction followed by the path. Figure 1-C shows this profile as a red continuous line, for the horizontal 0 degrees path, marked as a red line in image 1-B. If region is homogeneous with a well defined contour, profile's values will be very similar up to a point (the border) where there will be a change in the values. The strength of the change depends on how wide and soft the border is. If region is a tone down, values of $Profile_{\pi_{sq}}$ will be decreasing/increasing at a constant rate, that defines the strength or softness of the shade.

A suitable tool to study the behavior of a function is the derivative, that gives information about the variation at each point: in homogeneous regions derivative is around 0, while in inhomogeneities and borders, derivative has peaks. Whereas, in constant changes derivative will be constant at the magnitude of the change. Hence, we compute the derivative $Profile'_{\pi_{sq}}$, green dotted line in image 1-C, through the convolution with the derivative of the gaussian function that, in addition to derive, allows us to soften the function.

For our purpose, path's relevant information is in region's contour. It lead us to look for the peaks (maximums and minimums) in this derivative, since they represent points of maximum variation. In the original function, the profile, they correspond to gradient points that may be borders crossed by the path, as can be seen comparing the profile and its derivative in figure 1-C.

Here, we suppose that the first relevant gradient point corresponds to the border of the region. Therefore, to identify which of the peaks is the one we look for, we consider as relevant only the peaks whose magnitude is over a given percentage of the magnitude of derivative's maximum peak. The first peak verifying this condition is the one we take as the first relevant gradient point.

Let $peakM_{\pi_{sq}}$ be the maximum peak in the derivative, and $peakS_{\pi_{sq}}$ the selected relevant gradient point for the studied path, π_{sq}. In figure 1-C we mark $peakM$ as a black triangle, and $peakS$, that we suppose is in region's contour, as a red square.

Though in figure 1 we have shown, to reader's clarify, an example where $peakM$ and $peakS$ are in different position, if the chosen path's length is right according to the size of the region, both points coincide, meaning that the path only cross one border.

Once the peak is located, we approximate the slope around it, noted as $sl(\pi_{sq})$. We compute it as the slope of the rect (blue dotted line in figure

1-C) given by the two points delimiting the border (painted as blue circles in figure 1-C). These points, noted as r_l and r_r, are found as the crosses with zero in the derivative, on the left and right of $peakS$, respectively. Then, the slope is computed using the value in the profile of these points, as shown in equation 8:

$$sl(\pi_{sq}) = \frac{\triangle I(Profile_{\pi_{sq}}(r_r), Profile_{\pi_{sq}}(r_l))}{r_r - r_l} \tag{8}$$

where $\triangle I$ is the normalized gray level distance in section 2.2, and r_l and $r_r \in \pi_{sq}$.

At this point we have computed the slope of each path, and only have left to aggregate them to approximate seed surroundings's slope. This aggregation is done as the median, so the approximated slope of region surrounding the seed r_s, noted as $\bar{sl}_{r_s}$, is defined as the median value of the magnitudes of the slopes computed for the paths around the seed. The smaller this slope is, the more tone down-like the region is expected to be.

With it, we have characterized the area surrounding the seed with the approximated slope of its contour. Now it is necessary to establish a relationship between this slope and the homogeneity function used to model the region.

3.2 Homogeneity Function Parameter Approximation

As mentioned, regions are assumed to be homogeneous in color, so differences between regions are in their contours. Therefore, a suitable homogeneity function to model a fuzzy region $\widetilde{R_s}$ should fit region's contour. Intuitively, it means that the slope of homogeneity function used to model the fuzzy region, should be similar to the slope, $\bar{sl}_{r_s}$, computed for the region surrounding the seed.

Therefore, we assume a direct relation between both slopes, and use, to model a given region, the homogeneity function with the same slope as the region border. So now we look for a function that, from a slope value, gives the homogeneity function that models it.

In [4] we proposed the use of t-norms as homogeneity functions. Some of them, have a wide range of behaviors depending on their parameter value, corresponding each behavior to a different homogeneity function with different slope. Hence, we propose taking one of these parametric t-norms, and study the relationship between its parameter and the slope of the function. The selected function should be able to model the wider range of behaviors as possible: from soft shades with small slopes to regions with big slopes and crisp contours. One of the widest ranges is covered by Weber's t-norm, defined as in equation 9:

$$W(a,b,\lambda) = max\left\{0, \frac{a+b+ab\lambda-1}{1+\lambda}\right\} \tag{9}$$

where $\lambda > -1$. Hence, we center our study on it.

To perform the study, we choose a reference tone down, and analyze Weber's behavior along a path crossing it. The chosen reference tone down is a shade where the difference between one pixel and the next in the path, is 1 gray level, since this is the minimum tone down we may have, without repetitions of gray levels.

Results of evaluating Weber's t-norm on this path using different values of λ are shown in figure 2. This image shows different Weber's homogeneity functions, whose inclination do not present an homogeneous variation through all the range of λ: for $\lambda < 0$, small variations in λ induce noticeable changes in the slope of functions. For $\lambda \in (0, 1]$, increases in λ produce decreases in functions's slope smaller than for $\lambda \in (-1, 0]$. As λ augments, functions get closer and closer, up to around 10, where functions are almost the same though λ raises to ∞. In fact, the interesting range of values of λ is the one in $(-1, 1]$, since we saw in [4] that to model tone down regions we should use a value of λ around 0, and for regions with well defined contours λ should be near -1. It will be taking into account, doing in the study a thinner sampling in the interesting range of λ.

In the experiment we propose, we compute for each λ_i the corresponding homogeneity function. Then, we approximate the slope of each homogeneity function, noted as sl_i, through the slope of the rect between the point where function takes value 1, the first one in the path, and the point where it takes minimum value, as dotted green line shows in figure 2 for $\lambda = -0.9$.

With it, we have a set of points,

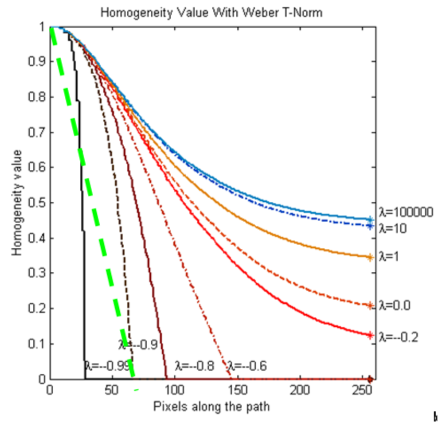

Figure 2: Weber's t-norm homogeneity functions for different λ, on the reference tone down. In black, the function for $\lambda_2 = -0.9$. Dotted in green, the rect to approximate its slope, sl_2.

$\{(sl_i, \lambda_i),\ i = 1..3000\}$, graphically represented in red color in figure 3.

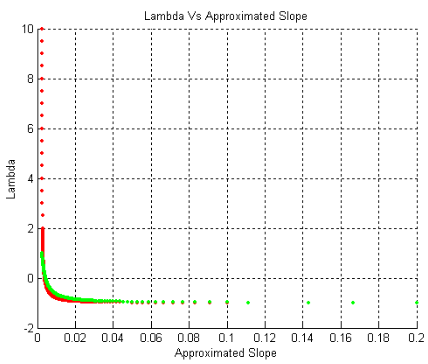

Figure 3: Graphic representation of obtained points (sl_i, λ_i), in red. In green approximated function, $\mathcal{F}(p)$.

Fitting a function for λ from sl

At the sight of this graphic, we look for a function:

$$\mathcal{F} : [0, 1] \rightarrow (-1, \infty) \tag{10}$$

that from a slope value, gives the associated value of λ corresponding to a suitable homogeneity function to model the region. To this purpose, we analyze red graphic in figure 3. As can be easily appreciated, graphic representation has two asymptote, one horizontal in $\lambda = -1$, because of it is the minimum value of λ, and another one vertical in $p = 0$, the minimum possible slope, corresponding to an homogeneous region. In addition, it may be appreciated that the real interval in which slope varies is approximately $(0.2, 0]$. After an analysis of graphic's characteristics, we concluded that function $\mathcal{F}(sl)$ may be approximated by a rational function on the absolute value of the slope, and displaced one unit down because of the vertical asymptote. In addition, to adapt the function to the vertical asymptote, the slope must be raised to some power, as equation 11 shows:

$$\mathcal{F}(p) = \frac{1}{s \cdot |p|^t} - 1 \tag{11}$$

where s is a scale factor and t is a growth parameter. After a process of parameter estimation, we have obtained the best approximation to the function in 3, in the sense of minimum square error, for $s = 976$ and $t = 1.27$. The approximated function is in green color in figure 3. Though values in the vertical asymptote are not well approximated, the interesting values of λ to model regions, the ones in (-1,1], have good approximations, with an error of 10^{-3} order.

An special consideration must be done about the parameter s. At the beginning of this study we have fixed a reference tone down. However, for any one it may not be a reference shade and would prefer to use a softer or harder tone down. Then, there are two options: repeat the experiment for that tone down, or use the s value as a proportionality constant between different reference tone downs. It is possible since changing the reference shade only produce a change in the scale of the graphic representation in 3. So we can view s as the parameter that allows us to adapt the function 11 to our concept of reference tone down.

So finally, we have obtained a function $\mathcal{F}(p)$ that, applied on the slope $\bar{sl}_{r_s}$ assigned to the seed r_s of a fuzzy region, gives the value of Weber's t-norm parameter λ_s, whose associated homogeneity function is suitable to model the fuzzy region.

4 Results

In this section, we present the results obtained applying to real images the proposed approximation to automatically select homogeneity functions. In all the experiments shown, we have chosen a set of eight paths covering eight homogeneously-distributed directions from the seed: from 0 to 315 degrees, with a step of 45 degrees. Each path is the straight line formed by the sequence of consecutive pixels in the corresponding direction. The length of the paths has been fixed to 60, and the function $\mathcal{PF}_{s_i}$ has been softened with a gaussian kernel of size 5 and $\sigma = 1$. The peak found as region border is the first one whose magnitude is over the 75% of the maximum peak in the derivative.

Figure 4-A to C show, in the first column, the original image with the seed point marked with a cross. Second column represents the eight paths on the intensity component of the original image. A small point in each path indicates the position of the first detected border. Finally, in the third column the membership computed for the region using Weber's t-norm with the λ obtained as in explained in previous section. In this image, white color represents maximum membership degree, while black means no membership.

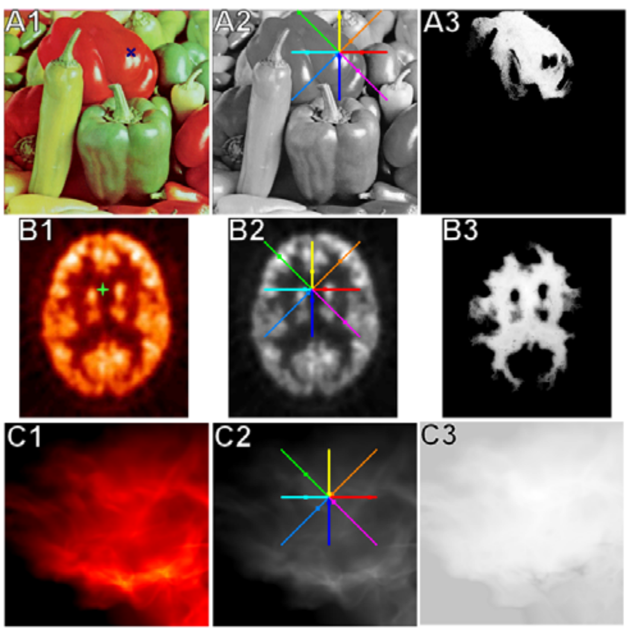

Figure 4: Results obtained with the proposed approach. 1: the original image with the seed in the cross. 2: the used paths. 3: the resulting membership image.

In figure 4-A1 to 3, we find the results obtained for the red pepper. As can be seen in image 4-A2, the point detected in each path as first relevant gradient point coincides in most of the cases with region's border, that is some cases is the border regarding the bright. The slope computed with this information, and the corresponding λ are in the first row of Table 1. With the Weber's homogeneity function for λ, we obtain a fuzzy region whose support covers almost the whole pepper's area, showed in 4-A3. In this image, there is a strong fall in the contour of the peppers, as the direct transition from white to black, with very few gray levels show. This is the expected result, since it is an homogeneous region with a well defined contour.

Figure 4-B1 to 3, shows that imprecision in contours produces a smaller slope value, resulting in a higher λ. In this case, soft changes between consecutive pixels in the paths, makes some paths, like green 135 degrees path in image 4-B2, detect the first gradient point in a different position than the one visually expected. However, again in most of the cases region's border coincides with the first relevant gradient point. Then selected homogeneity function produces a fuzzy region that fits the imprecision in central brain region's contour, as the softly decreasing gray levels (membership values) show in image 4-B3.

The last row in figure 4 shows a fuzzy region where, the computed slope is very small, resulting in a λ near 0. Therefore, the corresponding softer homogeneity function allows us to represent all the imprecision in the fuzzy region.

As Table 1 shows, in homogeneous regions with well defined contours, like pepper in figure 4-A, λ is near to -1, since it corresponds to a Weber's function with big slope, suitable to model fuzzy regions with great varying thin borders. It is obtained for slopes of 10^{-1} order, and down to around 0.05. The intermediate case is represented by image in 4-B, where the contour of the region is fuzzy. Therefore, it is necessary to use a softer homogeneity function, like the one obtained with a value of λ in approximately $(-0.8, -0.4)$ for a slope of 10^-2 order, as seen in the second row of the table. Finally, when regions are a tone down, like the one in 4-C, the value of λ increases and is around 0, while slope is of 10^-3 order, as can be seen in the last row of the Table 1.

Table 1: λ value obtained for each region.

Image	**Slope**	λ
A	0.066667	-0.968071
B	0.024183	-0.884258
C	0.005882	-0.303000

5 Conclusions

We have proposed an approach to automatically select the homogeneity function used in path-based image segmentation. This approach is based on approximating the slope of region's the border of, computing the slope of a set of paths starting at the seed and with different directions around it. With this slope, we obtain the value of the parameter of Weber's t-norm that gives an homogeneity function suitable to model the region, through the use of an approximated function. This function has been obtained by curve fitting to the set of points computed for a reference tone down.

Summarizing, results have showed that the proposed method serves as an approximation to select automatically an homogeneity function. An addition, when regions are homogeneous with well defined contours the value of slope is of 10^{-3} order approximately, and λ for their homogeneity function is near to -1. Meanwhile, in tone down regions, this parameter is near 0 and even above it if the tone down is very soft, corresponding to slopes of 10^{-3} order. In the rest of the cases, slopes of order 10^{-2}, the value of λ ranges between approximately $(-0.8, -0.4)$.

As future work, we shall study the use of all the color information in the computation if the paths and their slopes, through an the analysis of each color component separately or the use of a color gradient. In addition, the initial seed placement is an open topic, that may be solved applying the information obtained during the process explained here, to select useful seeds.

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