CONTENT BASED IMAGE RETRIEVAL USING A 2D SHAPE CHARACTERIZATION

J.M. Fuertes García
M. Lucena López
J.I. Gómez
Dept. de Informática
Escuela Politécnica Superior
Universidad de Jaén
Spain

N. Pérez de la Blanca
J. Fdez-Valdivia
Dept. de Ciencias de la Computación e I.A.
ETSI Informática
Universidad de Granada
Spain

Abstract

In this paper we present a Content-Based Images Retrieval (CBIR) system which uses characteristics of a region occupied by a shape and the outer contours of the same. The CBIR system is a two-stage image retrieval system: first, it extracts a subset of images which contains the shapes most similar to those contained in the query shape, based on the use of morphological characteristics of the objects considered. Then it indexes an extracted subset of shapes from the database in view of the similarity of the deformation of the outer contours of these shapes. For the first stage, we propose a geometric method of two-dimensional shape recognition based on the use of characterizations of those curves given by the morphological operators of erosion, closing, opening and dilation with structural elements of increasing size. The second stage of this system uses an invariant characterization of shapes based on a deformable model. In the experiments, we have illustrated that the CBIR system can accurately retrieve the most similar images from the database by using scaled, rotated, small deformed or mirrored sketched query images.

1. INTRODUCTION

In this paper we propose a system of indexation and query based on shape matching. Traditional shape description approaches include the extraction of a set of shapes and their representation as points of a multidimensional space. In related bibliography, we found description and retrieval systems based on shape. Representations using approaches to polygonal shapes and matching via polygonal vertexes [21]; shape matching using relaxation techniques in order to establish combinations of pairs of angles among shapes [4]; use of chains to represent and describe shapes [3][8]; comparing images using the Hausdorff distance [9]; or use of invariant moments, to represent characteristics of a shape with regard to transformations of translation, rotation and scale [12][17].

Previous techniques have made use of a single model and characteristics associated to 2-D shapes in order to describe objects, and one of the greatest limitations of using such simple models in shape description in content based image retrieval systems is the dependence on the domain of application.

This paper focuses on describing a method of image extraction based on the similarity between characteristics of the shape of images under consideration. Three proposals have been analyzed here in order to match objects contained in the database:

- Initially, an invariant characterization of finite sets based on mathematical morphology is studied. Comparing shapes with this method requires the prior extraction of features with a property of invariance with respect to certain geometrical transformations of the image [5]. In [22] we find a system of two-dimensional binary image recognition using characteristics given by morphological operators.
- Next, we are studied an invariant characterization of shapes based on a deformable model. This approach matches shapes according to the amount of effort it takes to deform an object and transform it into another one. References found in the bibliography related to the idea of object description using deformable models are between others [23][18].
- Lastly, we have proposed a 2D recognition system based on a combination of the above two. This approach is made up of two well-defined steps: first, from the entire database, the system extracts a subset of those objects which are most similar using the invariant characterization of finite sets based on the mathematical morphology given in Section 2, and then, it reorders those shapes retrieved using the criteria of similarity proposed by the deformable model described in Section 3. Basically, what this third approach to 2D shape recognition does is to join the main discriminating characteristics of previous techniques: discrimination of the area of the shape (mathematical morphology) and discrimination of the outer contour of the shape (deformable model).

The paper is organized as follows: Section 2 shows the morphological approach; Section 3 shows the scheme given by the deformable model; Section 4 presents the combined scheme of shape retrieval; Section 5 presents the results obtained by the three retrieval techniques showed and Section 6 presents the main conclusions of the work.

2. MORPHOLOGICAL FILTERS

2.1. Introduction

Using morphological models in shape recognition finds its development in the theory of random closed sets [13]. One image (background white) with a shape (foreground black) can be modelized as a point process or a random set; carrying out the point process is a random set. In this way we can define the random variable $N(E)$ as the sum of the number of points of set E.

The most obvious characteristic of a random set A is the probability that point x belongs to the set: $p = P(x \in A) = 1 - Q(\{x\})$, for any such point x, where $Q(x) = P(A \cap T) = \varnothing$. This value can be calculated by:

$$\hat{p} = \frac{\text{area}(A \cap W)}{\text{area}(W)}$$

whereas the area is accounted for by means of counting the points in observations A and $A \cap W$ over a window W. The calculation given in [22] and [13] may be applied to define necessary operators. Given two sets A and B we determine:

$$\check{A} = \{-x \mid x \in A\}$$

$$A \oplus B = \{x + y \mid x \in A, y \in B\}$$

$$A \ominus B = (A^c \oplus B)^c = \{x \mid B + x \subset A\}$$

and we define:

$$e_T = \frac{\text{area}\{(A \cap W) \ominus \check{T}\}}{\text{area}\{W \ominus \check{T}\}} \tag{1}$$

$$d_T = \frac{\text{area}\{(A \oplus \check{T}) \cap (W \ominus \check{T})\}}{\text{area}\{W \ominus \check{T}\}} \tag{2}$$

whereas the terms of the numerator of both expressions, using the Serra's notation [22], denote *erosion* and *dilation*. The dual operations denote *opening* and *closing* over disc T are given by, respectively: $(A \ominus \check{T}) \oplus T$ and $(A \oplus \check{T}) \ominus T$.

These operators are known as morphological transformations and they work on the observed part of A, $A \cap W$.

We can also define:

$$o_T = \frac{\text{area}[\{(A \cap W) \ominus \check{T}\} \oplus T] \cap V}{\text{área } V} \tag{3}$$

$$c_T = \frac{\text{area}[\{(A \cap W) \oplus \check{T}\} \ominus T] \cap V}{\text{área } V} \tag{4}$$

whereas V denotes the maximal set such that the opening of A by T is known within V from $A \cap W$.

Our goal is to describe the characteristic curves e_T, o_T, c_T and d_T as a function of T. We can clearly state that $e_T \leq o_T \leq c_T \leq d_T$

We can confirm that the four curves are monotonous in T, except in some cases due to the effect of the outer contours of the objects present in the scenes. The curves characterize the objects of the image according to a variation in the relationship of surfaces existing between what would be the area of the input object and that which is obtained by a morphological filter for each of the operators, when the radius of disc T varies.

2.2. Our approach

The question of shape recognition based on a set of characteristics may be redefined as the establishment of a similarity measure between an object considered and the objects in the database.

If we consider the shape and the simulation shown in Figures 1.a and 1.b for a disc T of increasing radius, the following points could be considered:

1. With respect to morphological operators, when the radius of structural element T increases, the area ratio given by the erosion and opening operators tends towards 0. On the other hand, and under the same circumstances, the area ratio given by the dilation and closing operators tends to be greater.

2. A window or frame of reference which allows us to fix the growth of dilation and closure should be delimited.

Both points require the definition of an adequate range of values for the used structural element T. In our approach, and in order to provide a normalized work environment, we applied an increasing range of values to T starting at 0 and increasing to whatever would be necessary for the elimination of the input object in the erosion process. In this way, we arrive at the representation of morphological curves such as those found in Figure 1.

Therefore, the result of this first stage of defining a shape recognition system is a vector of values representing the response from the shape of four morphological filters in a predetermined number of different diameters of T at equal distance.

2.2.1. Defining a measure of distance between the different characteristic vectors

The similarity of two shapes will be determined by the Euclidean distance between the vector of characteristics X of the input shape and vectors Z_i of

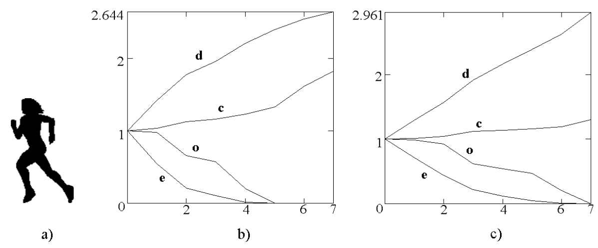

Figure 1: a) Original binary 2D shape. b) The simulation carried out for a circular structural element of increasing radius r ($r \in [1, 40]$) over the original image given a). c) Simulation carried out for a circular structural element of increasing radius r ($r \in [1, 32]$ over the original image a). Curves marked with **d** correspond to dilation, **c** to closing, **a** to opening and **e** to erosion.

those shapes contained in the database:

$$d(X, Z_i) = \sqrt{(X - Z_i)^T (X - Z_i)} \tag{5}$$

The measure of distance given in the above equation (5) allows us the indexation of the different shapes considered in the database.

2.3. Supported Invariances

The proposed procedure for measuring similarity supports an invariance over three different transformations:

1. Invariance to a *scaling transformation* is assured by the normalization mechanism that establishes a previous shape scaling step which assures that the shape disappears completely when the erosion operator is applied with a structural element T of a maximum predetermined size.

2. Invariance to a *translation transformation* is also assured since the magnitude itself measured in the morphological curves attends to a relationship among surfaces.

3. Invariance with relation to a *rotation transformation* is also assured as long as the structural element T used in the morphological operators is a centered disc.

2.4. Extension of the Approach to Non-isotropic Elements

This paper has included another experiment via the consideration of non-isotropic structural elements such as rectangulars positioned on their respective centers of gravity.

With respect to the questions of invariances supported by this approach and mentioned in Section 2.3, several points can be made with regards to using non-isotropic structural elements:

- All those considerations made for the invariance approach under transformations of scale and translation are still valid.
- The calculations used to assure the invariance with respect to rotation must be modified, as follow.

2.4.1. Invariance to Rotation over Non-isotropic Structural Elements

If we want to preserve an invariance to transformations of rotation, a study would have to be carried out on the morphological curves bearing in mind that the structural element rotates $360°$.

Because this procedure considerably increases the computational cost of the process, we use a discretization of the first two quadrants. Since the structural elements are positioned on their center of gravity, discretization will affect the third and fourth quadrants equally.

Figure 2 shows two shapes, rotated $90°$ to each other and their corresponding morphological curves obtained while considering four different angle steps for a rectangular structural element positioned on its center of gravity of a maximum size of 16 pixels wide by 32 pixels high.

The curves do not differ in the same sequence, but the groups of morphological curves have shifted, so the proposed solution is very simple:

1. Determine the displacement produced by the groups of curves.
2. Apply a measure of distance between the new vectors obtained whose elements have been previously corrected by possible displacement due to transformations of rotation.

3. DEFORMABLE MODELS

With respect to contour of the 2-D shape, we can define a property of deformation, between the object in question and the analyzed correspondent, which

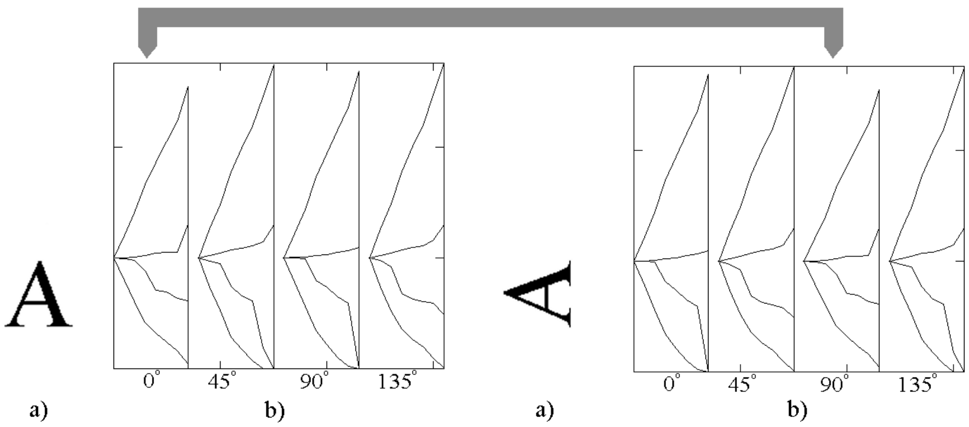

Figure 2: Simulation carried out with a rectangular structural element positioned over its center of gravity, of a maximum size of 16 pixels wide and 32 pixels high. a) the original images, b) the corresponding graphics.

allows us to match objects in the database. This leads to the use of a deformable model. We will have to define some type of measure which allows us to evaluate the *effort* or *energy of deformation* which we have to apply to a given contour in order to adapt it to another.

Different ways of modeling deformations are described in the bibliography [2] [11] [20] [1] [7] [10] [6].

We have used the deformable model given by Nastar in [14] [15] [16] which is decribed briefly below.

3.1. Mode Analysis

The system's equation of movement is given in the form of a matrix of $3N$ dimension by:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}(t) \tag{6}$$

whereas $\mathbf{U}$ is a vector which stores the displacement of nodes, $\mathbf{K}$ is the system's stiffness matrix, $\mathbf{C}$ = diagonal (c_i) is the buffer matrix, $\mathbf{M}$ = diagonal (m_i) is the diagonal masses matrix and $\mathbf{F}$ is the image of force which contains the object attracted by the contour images. Matrices $\mathbf{C}$, $\mathbf{M}$ and, in particular, stiffness $\mathbf{K}$ matrix, are constant, which is why they should not be recalculated in each iteration. Equation (6) gives the formulation based on the use of finite elements of the deformation process.

A method of mode analysis based on frequencies will be used for this.

In the line of providing a solution to the balance equation (6), we can transform the problem by changing the base: $\mathbf{U}(t) = \mathbf{P}\tilde{\mathbf{U}}(t)$, with $\mathbf{P}$ the squared N order non-singular matrix transformation which should be calculated and $\mathbf{P}\tilde{U}$ is the vector of order N, known as the *vector of generalized displacement.* The equation of motion in the new base is given by: $\tilde{\mathbf{M}}\ddot{\tilde{\mathbf{U}}} + \tilde{\mathbf{C}}\dot{\tilde{\mathbf{U}}} + \tilde{\mathbf{K}}\tilde{\mathbf{U}} = \tilde{\mathbf{F}}(t)$

In practice, an effective transformation is obtained by using the solutions to the equation of balance, assuming displacements of free vibration with $\mathbf{F}(t) = 0$. Obviously, if the system is to balance itself, $\mathbf{C}$ should be 0 so there is no loss of energy, thus resulting in the equation $\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = 0$

The solutions to this equation are harmonics of the type $\mathbf{U} = \phi \sin \omega(t - t_0)$, where ϕ is a vector of order N indicating the extent of the vibration, t is the temporary variable t_0 is a constant of time and ω is the frequency of the vibration of vector ϕ.

If we substitute the solution in harmonic form in the equation $\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = 0$, the result will be a generalized eigenproblem from which we can determine ϕ and ω: $\mathbf{K}\phi = \omega^2\mathbf{M}\phi$

The solution to this eigenproblem ([19]) is made up of $3n$ eigensolutions.

If we define a matrix $\mathbf{\Phi}$ whose columns are eigenvectors ϕ_i and a diagonal matrix Ω^2 which has the ω_i^2 eigenvalues on its diagonal, then the $3n$ solutions can be written as follows:

$$K\Phi = M\Phi\Omega^2 \tag{7}$$

Matrix $\mathbf{\Phi}$ will be an appropiate transformation matrix (known as P), and therefore the expression is as follows:

$$\mathbf{U}(t) = \mathbf{\Phi}\tilde{\mathbf{U}} = \sum_{i=1}^{N} \tilde{u}_i(t)\phi_i \tag{8}$$

where ϕ_i is the $i-$mode, $\tilde{u}_i$ is the amplitude and ω_i is its frequency.

In fact, an approach to the nodal displacements $\mathbf{U}(t)$ as $\hat{\mathbf{U}}(t)$ is done, the sum of p modes with lower frequency, being $p \ll N$.

Vectors $(\phi_i)_{i=1,\ldots,p}$ form a *reduced modal base of the system* and allow a closed form solution through the selection of modes with lower frequency.

3.2. Analytical Calculation of the Proper Vibration Modes

Once the elastic structure has been defined, it is necessary to present an analytical expression which allows the calculation to be carried out. Although it is possible to directly use numeric procedures, we must take into account that the size of the used matrixes could be long size for the most interesting case of 3D borders (surfaces). Nastar ([14]) proposes a calculation to determine the analytical expression of the modes and, thus, considerably reduce the costs of its computational calculation.

In the case of a closed curve, the eigenvalues ω^2 can be defined by the dispersion equation:

$$\omega^2 = \frac{4K}{M} \sin^2\left(\frac{p\pi}{n}\right) \tag{9}$$

We should point out the fact that if N is odd, $\omega_0^2 = 0$ is the only simple eigenvalues and the rest are all double eigenvalues. If N is even, $\omega_0^2 = 0$ and $\omega_{n/2}^2 = 4K/M$ are the only simple eigenvalues, the others will be double eigenvalues.

For the simple eigenvalues, the expression of the associated eigenvector is given by :

$$\phi = \left[\cos\frac{2p\pi}{N}, \ldots, \cos\frac{2p\pi n}{N}, \ldots, \cos\frac{2p\pi N}{N}\right]^T \tag{10}$$

where $p \in \mathcal{B}(N)$, the *first Brillouin zone* [14].

$$\mathcal{B}(N) = \begin{cases} \left\{-\frac{N-1}{2}, \ldots, \frac{N}{2}\right\} & \text{if } N \text{ is even} \\ \left\{-\frac{N-1}{2}, \ldots, \frac{N-1}{2}\right\} & \text{if } N \text{ is odd} \end{cases} \tag{11}$$

For double eigenvalues, we obtain a couple of orthogonal eigenvectors given by:

$$\begin{cases} \phi = \left[\cos\frac{2p\pi}{N}, \ldots, \cos\frac{2p\pi n}{N}, \ldots, \cos\frac{2p\pi N}{N}\right]^T \\ \phi' = \left[\sin\frac{2p\pi}{N}, \ldots, \sin\frac{2p\pi n}{N}, \ldots, \sin\frac{2p\pi N}{N}\right]^T \end{cases} \tag{12}$$

3.2.1. Deformation Spectrum and its Application

As seen in the beginning of Section 3, the idea is to define an object of a deformable model in order to adapt it to another, allowing us to measure the *energy used in the process as object similarity.* Once we have an analytical expression of the vibration modes of the nodes situated upon a closed curve, we will define a representation mechanism of the modal width value as a function of the shown mode: $\tilde{u}_i(t) = f(i)$

This representation will be called modal spectrum [14]. Basically, it describes which modes and in which quantity the different modes are excited in order to alter a shape until it matches another one. Thus we can obtain an indicator of the quantity of *deformation energy* needed for the process:

$$E_{deformation} = \frac{1}{2}\mathbf{U}^T\mathbf{K}\mathbf{U} = \frac{1}{2}\sum_{i=1}^{N}\omega_i^2\tilde{u}_i^2$$

Two deformations will be *similar* when their corresponding displacement fields $\mathbf{U}_1$ and $\mathbf{U}_2$ are similar, except the rigid transformations, which contain zero deformation energy. In other words: *two deformations will be similar when their corresponding deformation spectra are similar.*

3.3. Similarity Measure Between Two Given Deformations

In this section we describe all steps to obtain deformation spectra of the different shapes used and to establish the similarity among them.

Since it is necessary to define a vector with characteristics that allows us to determine the deformation of a given shape, we will have to define a shape prototype, common to all those shapes chosen, against which, the deformation can be calculated. The circular shape of a r radius selected *a priori* has been chosen as a prototype.

Steps algorithmically are:

1. Scaling the shape analyzed by a factor so that its maximum radius coincides with the r radius of the circular shape.
2. Superposing the shape already escalated with the contour of the r radius circle, so the shapes intersect in at least in one point.
3. Sub-sampling the contour of both shapes in a number of points, fixed *a priori*, that will be the nodes shown in later calculations.
4. Calculating the modal deformation coefficient of the analyzed shape to the prototype circle used.
5. Sorting the eigenvalues and their associated eigenvectors in increasing order by eliminating the modes corresponding to rigid motion.

Finally, the algorithm takes the deformation spectrum of a shape given to another circular one. This will have to be done with the other shapes, under the same conditions of r radius and number of points of sub-sampled contours. In other words, all deformation spectra of the shapes will be obtained. Then, we would only have to establish a distance between the deformation spectrum of the shape given as input and of the shapes contained in the database to establish a similarity rate. As distance, we can use any Euclidean rule as:

$$d(D_1, D_2) = \frac{1}{p}\sqrt{\sum_{i=1}^{p}(\tilde{u}_i(D_1) - \tilde{u}_i(D_2))^2} \tag{13}$$

where D_1 and D_2 are the deformation labels for each of the modes shown.

3.4. Supported Invariances

The similarity measure presented, supports an invariance for three different transformations: Scaling, translation and rotation.

The invariance to scaling transformation is assured by the normalization mechanism described in the algorithm described in Section 3.3.

The invariance to a translation transformation is also assured due to the fact that upon considering isolated objects that are scaled (point 1 of the similarity measure algorithm presented in Section 3.3), the shape's position within the image has no influence.

The invariance to a rotation transformation is also assured since the method used only responds to deformations that must be applied to pairs of points (prototype - object), so the possible rotation transformations will not affect the scheme.

4. COMBINATION OF MORPHOLOGICAL FILTERS AND DEFORMABLE MODELS

The 2D shape recognition schemes shown in Sections 2 and 3 describe, with different approaches, two similarity calculations between a given shape and those contained in the database.

To establish the matching, the approach based on the morphological filter uses area features of the 2-D shape under consideration. On the other hand, the approach based on the deformable model uses the outer contour of the analyzed shape. According to results obtained in Section 5, the characteristics shown by both techniques suggest, in fact, that both techniques should not be understood as alternatives to be used, but as complementary techniques. Therefore, in order to match two shapes, both the characterization of the area of the shape, and the deformation energy of its outer contour and the data must be taken into account to index the result of the query.

4.1. Proposed Approach

Establishing similarity/matching between an input shape and the shapes contained in a database will be carried out in two different stages.

1. Using a given shape, as an input to the recognition system, we intend to index the shapes contained in the database comparing its similarity to the query shape. Thus, we will extract the $n-$first most similar shapes to the input prototype. The first extract will be carried out by using the morphological filtering technique shown in Section 2. In this way, we characterize the similarity by using the information supplied by the area filled by the shape.

2. Once we have the subset of the database shapes most similar to the one given in the input, a similarity measure is again established between the input shape and the subset of shapes retrieved in the first phase. To calculate the second similarity measure, a deformable model technique is used, as shown in Section 3. Thus, we characterize the similarity by using the information supplied by the shape contour.

The proposed approach involves linking both techniques initially described. The first one works as a database consultation filter to define a subset of shapes that the second one will order according to the outer contour of the shape.

5. EXPERIMENTAL RESULTS

In order to show the results obtained with the proposed 2D shape recognition technique, we have used a database containing 161 2D simple shape images, which include: squares, rectangles, triangles, ellipses, pentagons, hexagons, characters, animals, airplanes, cars and silhouettes.

Two different experiments were carried out with the technique proposed in Section 2. Both use the same morphological operators (erosion, dilation, opening and closing) and a structural element whose size is large enough for those objects being characterized to disappear completely when the erosion operator is applied:

1. *Use of an isotropic structural element.*

2. *Use of non-isotropic structural elements.*

For the technique proposed in Section 3, experiments carried out have modified two elements:

1. *The number of modes considered in the modal deformation space.*

2. *The number of subsampled points considered in the outer contours of those query shapes.*

For the technique proposed in Section 4, in the morphological filter phase, in those experiments carried out, we have used a rectangular structural element, positioned on its center of gravity with a maximum size of 2 pixels wide by 32 pixels high, considering 16 values of the morphological curves and 8 rotation angles for the structural element, which in this way cover 180°. In the second stage, we have fixed a maximum prototype size equivalent to a circular shape with diameter of 128 pixels. Regarding the number of points of the contour shown, 128 points have been sub-sampled. Both options have been carried out.

We have made three experiments: Morphological Filtering and Modal Deformation, showing the result of the techniques explained in Section 2 and 3, and finally the Combination of both techniques.

Figures 3 and 4 shown the results obtained by using the different experiments in some of the families considered.

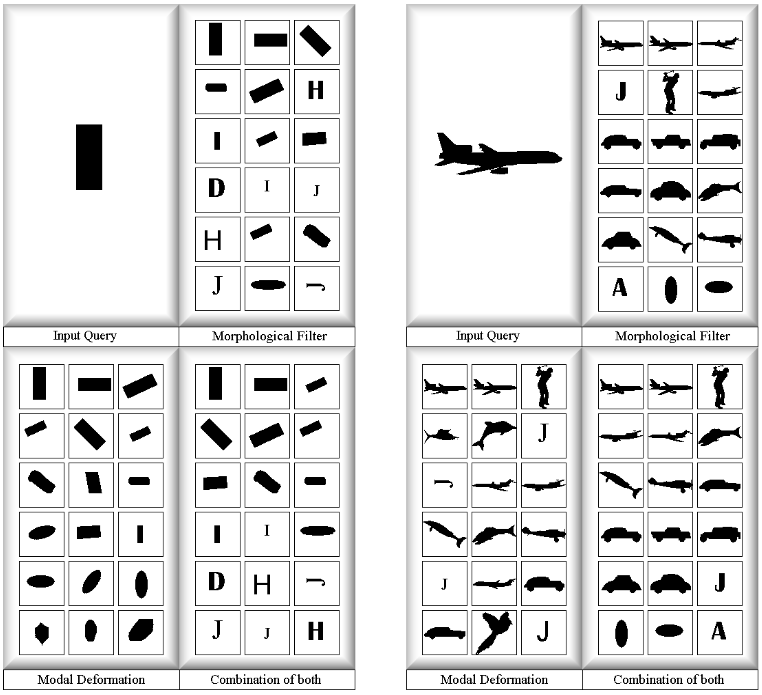

Figure 3: Indexing of the first 18 shapes retrieved by using the three recognition techniques described.

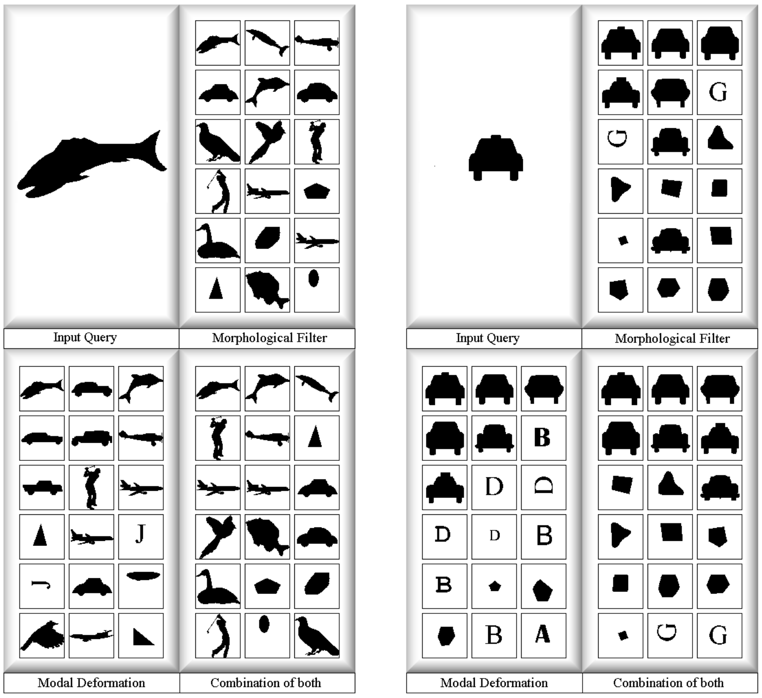

Figure 4: Indexing of the first 18 shapes retrieved by using the three recognition techniques described.

6. CONCLUSIONS

This paper has dealt with the question of indexing shapes contained in a database beginning with the concept of similarity. For this purpose, several discriminating characteristics have been described within the different shapes which allow us to calculate their similarity, from a shape recognition process.

We propose two basic 2D shape recognition techniques: The first is based on the morphological filtering process, in which, via the use of operators such as: erosion, dilation, opening and closing, a geometrical recognition scheme covering the studied area is described. The second shows an invariant characterization of shapes via a deformable model. Here, we measure the necessary quantity of energy which need be applied to the outer contour of an object in order to transform it into another one.

In addition, a third technique is shown, which combines the two aforementioned techniques. This technique makes use of the morphological scheme in order to extract a subset of those most similar images from the database. The energy measure of the deformable model is then applied to index the resulting subset.

Experimental results have been obtained for queries made for several shapes varying the used structural element and the number of modes and subsampled points of the same allows several points to be made:

1. 2D shape characterisation using the morphological technique shown in Section 2 is appropriate when the set of shapes to be retrieved contains morphologically different elements in the areas occupied by the same: inner holes, re-entering or salient elements, etc.

2. The use of a deformable model shown in Section 3 is appropriate for retrievals over shapes whose outer contour is the most discriminating characteristic.

3. The use of both techniques mentioned in Section 4 on very diverse 2D shape databases, allows, according to results obtained in experimentation, for the retrieval of shapes which are most similar. We must bear in mind that this technique uses characteristics of both the area occupied by the shape and the outer contours of the same.

Experimental results obtained from the database are shown for each of the different techniques. Results indicate how indexing generated by the third technique is the most appropriate for the input prototype shape.

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