Reflections on an old problem: that of preserving the logical forms

Adolfo R. de Soto[1][0000-0002-3478-4590], Alejandro Sobrino[1], and Enric Trillas[3]

[1] School of Industrial, Computer and Aeroespace Engineering, University of León, León, Spain
adolfo.rdesoto@unileon.es
[2] Faculty of Philosophy. University of Santiago de Compostela, Spain
alejandro.sobrino@usc.es
[3] etrillasetrillas@gmail.com

For most, even illustrated and well educated people, "reasoning" consists in deducing, rationality consists in deductive reasoning, and irrationality in any other way of reasoning. Of course, this kind of "ideology" is deeply mistaken since in more than the 75% of the times people ordinarily reasons by means of abductive, speculative, or analogical forms that are, essentially, conjectural forms of reasoning. For instance, in science the establishment of hypotheses is one of its main objectives, and it is often reached by analogy with some previously known and similar problems. It should be pointed out that all those types of non deductive reasoning do not show the typically deductive property of monotonicity, that is, the growing of the number of conclusions when that of premises grows. On the contrary, the number of conclusions either can decrease, or there is no any foreseeable property of monotonicity; that is, these types of reasoning are either anti-monotonic, or just non-monotonic. Rationality is shown when arguing by means of reasons, when the conclusions of reasoning are reached from the best available previous information or premises, and through the most convincing than possible methodology that can support them in the base of the given premises. Nobody will say that a scientist that makes an apparently well founded hypothesis on something, is irrational by just this fact. Hence, the branch of Philosophy known as Pragmatics cannot be reduced to only taking into account deductive reasoning; to confuse inference with deduction. This even considering, in a first place, that formal deduction is the safest way of searching for conclusions, but not ignoring, in a second place, that the word "deduction" is just understood as "formal deduction" that strongly requires to be done in a given formal framework. A formal framework like it is a Boolean algebra for the classical propositional logic, or the orthomodular lattice of the closed linear subspaces of an infinite dimensional Hilbert Space for quantum physics logic. Such a formal framework almost never is at our hands when doing ordinary, everyday, or commonsense reasoning. What in ordinary reasoning is sometimes called "deduction", and although it should be necessarily monotonic, is not exactly "formal deduction", and yet deserves a comprehensive study. For instance, it is not clear if in commonsense deduction, either the premises are to be considered as conclusions, or the set of conclusions of the conclusions is coincidental with this last.

One of the topics that Pragmatics should consider is the analysis of the play of the logical constants in the language that, thanks to some of them, allow to establish the so-called logical forms like they are, for instance, the associative and the commutative laws of the constant "and". Precisely, since "and" is not showing all the same properties in common discourses than in formal ones, not only the use of the old term "constant" does by submitted to doubt, but also that of "logical constant", especially if Pragmatics is viewed from the use of natural language and ordinary reasoning. For example, the statements “She entered into the room and cried”, and “She cried and entered into the room”, cannot be taken as identical and, hence, the commutative form p and $q = q$ and p is broken. When time intervenes between p and q, it is not possible to always keep the commutative law of the linguistic “and”. All that, places the problem known by the “preservation of the logical forms” into a new perspective, and in particular it seems to show that the term “logical constants” does be currently avoided.

To illustrate such a new perspective it is a good example to consider the case of the linguistic use of imprecise predicates, something that is pervasive in common speeches. As it is known the only currently known way for representing imprecise predicates is that facilitated by fuzzy sets. If $([0,1]^X, \cdot, +, ')$ is a Basic Fuzzy Algebra (BFA) in which the imprecise linguistic expressions are represented, the classical law (or form) of “perfect repartition”, $A = A \cdot B + A \cdot B'$, is not generally verified except if, for instance, the BFA is isomorphic to that given by the triplet of functionally expressible connectives $\cdot = W$, $+ = \text{prod}^*$, and $' = 1 - id_{[0,1]}$, with $W(a,b) = \max(0, a+b-1)$, $\text{prod}^*(a,b) = a+b-a\cdot b$, and $a' = 1-a$. And it should be pointed out that the standard algebras of fuzzy sets isomorphic to the standard algebra $([0,1]^X, W, \text{prod}^*, 1 - id_{[0,1]})$ are neither distributive, nor dual, nor verify the laws of non-contradiction and excluded-middle. That is, in the setting of fuzzy sets, perfect repartition, duality, distributivity, non-contradiction, and excluded-middle, are not coexistent logical forms. Of course, this example places the preservation of logical forms in a different perspective than that usual in Philosophy. Lets notice again that perfect repartition follows, in the case of precise predicates that is, in the formal setting of Boolean algebras, from distributivity and excluded-middle: $A \cdot B + A \cdot B' = A \cdot (B + B') = A \cdot 1 = A$.

This formal reasoning shows that in the case of quantum logic, where distributivity is known to not holding, the “form” $A = A \cdot B + A \cdot B'$ cannot be used for all A and B. Hence, both in the precise speeches of quantum logic, and in the imprecise ones of common language, it is not generally possible to freely use this form. In the second case, it is possible to use the form, but under the mandatory condition of working in a BFA allowing it, and like it is, for instance, $([0,1]^X, W, \text{prod}^*, 1 - id_{[0,1]})$. Hence, the current use of the constants “and”, “or”, and “not”, is essential for taking into account the preservation of the “perfect repartition” form.

Obviously, these argumentations place the problem of the preservation of logical forms in a new perspective. The ordinary reasoning shows this problem in multiple examples and Fuzzy Sets Theory has allowed to study some of those cases of lack of preservation of some common properties of the logical forms depending on the modeling of the logical operators used. However, a representation of logical operators closest to ordinary reasoning remains open and Fuzzy Logic is the best known field to continue deepening in this field.