A GLANCE AT SYMMETRY, IN FUZZY LOGIC, AND LANGUAGE

Enric Trillas (University of Oviedo. Oviedo, Asturias; Spain).

ABSTRACT

Relevant in both Nature and Art, symmetry also appears in Science where it is viewed an interesting and important feature of what is studied, and that often conducts to results considered 'beautiful'; symmetry and beautifulness are not fully separable. Nevertheless, it should be pointed out that in Science not all shows symmetry; for instance, there is neither symmetry between the fundamental concepts of 'proving' and 'falsifying', nor between 'premise' and 'conclusion'; anyway, in this last case the symmetry can be seen as 'h is a hypothesis for p ⇔ p is a consequence of h', where p is changed by h, and 'hypothesis' by 'consequence'. This is a basic idea for viewing symmetry as some 'invariance' between objects, either physical or virtual, when they are submitted to some transformation.

The beautifulness associated to symmetry not only arises in Nature and Art but also in Science; for instance, crystal symmetry in Geology, molecular symmetry in Chemistry, and the importance of Einstein's symmetrical laws in Physics that probably inspired Emmy Noether's famous theorem on the preservation of some magnitude for all symmetry. In Mathematics, and apart the study of the groups of symmetry, mathematicians appreciate some proofs of theorems as beautiful when they are deductively short and compact a lot of specialized knowledge. Also some books of mathematics are considered beautiful in themselves like it is, for instance and among others, the famous Emil Artin's 'Geometric Algebra' mainly because of its elegant proofs. Also some computer done graphics are considered beautiful, as it is that of the Menger's 'sponge', and those coming from Benoît Mandelbrot's Fractal Theory like it is, for instance, the well known Mandelbrot set, etc.

Symmetry refers to the existence of something that remains immune to change, that shows some invariance; for instance, the disjunctive operator representing the language's 'or' shows symmetry if it is commutative, and the so-called 'implication' representing conditional statements, is not symmetrical in general. Provided the conditional statement is understood by the conjunctive form $p \rightarrow q = p \cdot q$, then its symmetry depends on the commutative law of the conjunction $(\cdot)$, a property that not holds universally in plain language since time almost always intervenes in it; like the associative property depends on the possibility of changing commas. It can be distinguished the symmetry in definitions, and the symmetry in properties and in formulas. In fuzzy logic, seen as the study of both the mathematical modeling of the imprecision and not random uncertainty appearing in plain language's descriptions, there are

also some instances of symmetry, but, in general, what is in this 'natural phenomenon' not always shows symmetry.

The paper behind this Abstract tries to analyze some aspects appearing in ordinary reasoning and fuzzy logic related with either symmetry, or with its absence that, notwithstanding, in some particular cases can become symmetrical as they are, for instance, the linguistic negation, the law of perfect repartition, the additive law of measures, etc. In any case, the differences between what is supposed in a mathematical model and what can be checked in the reality cannot be forgotten; any reality is but an 'observed reality'. Anyway, if what is in a fuzzy model is not always symmetrical, and since these models represent linguistic statements affected by imprecision and uncertainty, it can be concluded that, in general, neither plain language, nor commonsense reasoning, show the same level of symmetry shown by the specialized language and the deductive reasoning allowing mathematical proofs. Symmetry can appear in a model but without appearing in the reality it tries to represent. If symmetry is pervasive among the concepts concerning the logical calculus with precise concepts, it reduces considerably when passing to the calculus with imprecise concepts given by the algebras of fuzzy sets. A reason for it lies in the fact that the almost perfect mathematical model Boolean algebras offer for the first, disappears within the second in which the mathematical model is not unique, and in all of them the big number of laws that Boolean algebras enjoy reduces to a less number of them causing the loss of some basic classical laws.

Often, the exhibit of symmetry is due to the adopted mathematical model attending at how it is presumed the reality submitted to study. It is not fully dislike with the fact that, even if people see the world in colors, the world itself is uncolored; are the human eyes that color the world through its way of perceiving the light's wave lengths. Anyway, mathematical models are a good enough form of looking white over black at the problems and, for what concerns the case of imprecise concepts, it should be presumed a minimal number of laws among those holding in the crisp calculus, with the goal of truly approaching the reality of plain language and ordinary reasoning in whose several parts some of those laws hold, but not in other parts. At the end it cannot be surprising that in plain language and commonsense reasoning, symmetry is not always present; it is not at all a bizarre phenomenon. For instance, the beautiful and marvelous symmetrical snow's crystals, are not seen directly at nude eye and appear when looking them by means of a microscope. Not necessarily symmetry appears in the 'reality', but in the 'observed reality' the only that we can know, and that often it can be through an 'instrument' like it is a microscope, or a theory that permits investigation with the help of some formal, mathematical model trying to represent the presumed reality; Science is for approaching the ultimate and usually hidden reality. If ultimate realities are what they are, symmetry appears in its scientific representations. At the end, all that we can know are but local representations created by the filters used to observe, or construct them.